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Time–Weissenberg number superposition in planar contraction microchannel flows

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ABSTRACT

The flow of viscoelastic fluids in a micro-fabricated 4:1 planar contraction channel was studied through flow visualization. As the Weissenberg number (*Wi*) increased, the flow developed from a Newtonian-like to vortex growth, and the transient start-up flow at high *Wi* was found to experience all the steady patterns at lower *Wi* flows. The flow sequence was different depending on the fluids and channel dimensions, however, in all the cases we could reach, the steady patterns at a low *Wi* flow could be matched 1:1 with the transient patterns at a high *Wi* flow. The plot of *Wi* and time when the two sets (transient and steady) were matched showed a clear functional relationship, from which the time–Weissenberg number superposition could be confirmed.

1. Introduction

The flow of viscoelastic fluids in contraction geometry appears in many applications, from traditional technology such as polymer processing to emerging technology such as coating and printing. There have been many studies on flow dynamics in contraction geometry, and the most characteristic feature seems to be vortex enhancement. When the elasticity of the fluid or the flow rate is low, there is no significant flow pattern other than smooth streamlines with no flow separation. However, with an increase in flow elasticity or an increase in the shear rate, vortex appears and its enhancement process has attracted much attention [1,2]. The effect of the contraction ratio on flow behavior was also explored using fluids of low to high elasticity [3]. However, most of the studies on contraction flow were performed using macro channels (i.e. channel size was on the order of millimeters or larger). Although macro channels have advantages in dealing with highly viscoelastic fluids, it is difficult to reach a high shear rate due to their large size. In addition, because the increase in flow rate also increases the Reynolds number (*Re*), it becomes hard to distinguish whether flow instability comes from elasticity or from inertia. However, it is possible to obtain high deformation rates while maintaining a low Reynolds number if a micro-fabricated channel is used. In other words, a high Weissenberg number (characteristic time of the fluid multiplied by the characteristic shear rate) flow could be explored while maintaining a low Reynolds number due to the small scale length of the microchannel. There have been many studies on the viscoelastic flow with a low Reynolds number using microfluidic devices [4–6]. Simulations were also performed in an effort to understand the dynamics of a highly elastic flow with low Reynolds number [7]. It was demonstrated by both experiment and simulation that a vortex is formed with a high Weissenberg number even when there is no inertial force. However the mechanism is not fully understood because the flow heavily depends on the fluid and channel dimension. In the contraction flow for the macro-size, also in numerical simulation, a lip vortex often appeared as the Weissenberg number increased. However, in a planar contraction channel whose width is on the order of tens or hundreds of micrometers, there have been many cases where a lip vortex did not appear and a unique flow pattern called a divergent flow appeared. A divergent flow is the flow in which the location of the maximum flow velocity is not right above the point of the contraction but is moved upstream due to fluid elasticity, resulting in a distortion of the flow. This phenomenon was also demonstrated in a simulation using the upper-convected Maxwell (UCM) model [8]. There have been many attempts to explain such a diverse flow development, for example, by changing the Elasticity number, which is the ratio between the Reynolds number and Weissenberg number [9].

Most of the previous studies have focused on the flow patterns at steady states, while experimental studies on the transient behavior have rarely been performed. There are some simulations about transient of start-up flow in contraction or constriction (contraction-extension) geometry. Bishko et al. calculated how

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vortex formed when the viscoelastic fluid flows though a 4:1 contraction geometry at various *Wi* [10]. There also exists a study on the transient behavior, in which the center of the vortex changes its motion when the pressure drop changes to slow growth in the start-up flow [11]. Kim et al. performed a numerical study and reported that the transient flow at a certain Weissenberg number experienced all the steady flow patterns at lower Weissenberg numbers, and suggested the principle of superposition between time and the Weissenberg number. In their graph of the time–Weissenberg number superposition, the slope changed at the point where the lip vortex and the corner vortex merged [12].

In this study, we investigated the flow dynamics in a micro-fabricated 4:1 contraction planar channel by changing the fluid elasticity and channel dimension. All the transient flow patterns at the initial high Weissenberg number flow were compared and matched to the flow patterns at steady states at lower Weissenberg numbers. By doing this, we tried to prove whether the time–Weissenberg number superposition which was proposed by numerical simulation could be a principle involved in the complexity of this flow or simply just the result of numerical artifact.

2. Experiments

2.1. Fluids

In this study, poly(ethylene oxide) (PEO, Aldrich; MW = 2×10^6 g mol^{-1}) solutions of 0.3 wt%, 0.7 wt%, 1.0 wt% were used. The steady shear viscosity of each solution was measured using a strain controlled rheometer (ARES) with 60 mm parallel plates. The viscosity curves as a function of the shear-rate are shown in Fig. 1. The zero-shear viscosity and the relaxation time of each solution are provided in Table 1. The relaxation time was measured using a capillary breakup extensional rheometry (CaBER) [13].

2.2. Microchannel

Four contraction channels, with schematics shown in Fig. 2, were made with the dimensions presented in Table 2. W_u is the width of the upstream channel; W_c is the width of the downstream channel, and *h* is the height of the channel. The channel was constructed using poly(dimethylsiloxane) (PDMS), with a mold made with SU-8 photo-resist and high-resolution chrome mask [14,15]. SU-8 was spin coated with a uniform thickness onto a silicon wafer, and the mold was patterned by soft-lithography of the chrome mask. PDMS was poured over the pattern and the transparent PDMS channel was obtained after heating in an oven. The experiments were performed using these channels attached to a cover glass which was thinly coated by PDMS at a thickness of 5–7 μm.

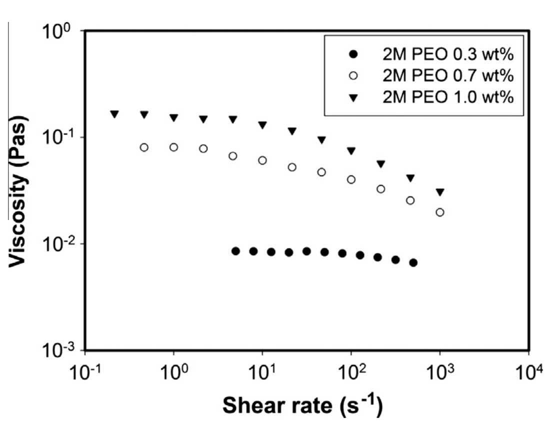

Fig. 1. Viscosities of the PEO solutions as a function of the shear rate.

Table 1
Zero-shear viscosity and relaxation time of each solution at 25 °C.

	2 M 0.3 wt%	2 M 0.7 wt%	2 M 1.0 wt%
η_0 (Pa s)	0.009	0.080	0.170
λ (ms)	14	35	44

The ratio of the PDMS to the curing agent (Sylgard 184B) was 10:1. Because we attached the PDMS coated cover glass to the PDMS channel, the adhesion was good, and all the channel walls had the same hydrophobicity.

2.3. Flow visualization

0.03 wt% of 1.0 μm carboxylate-modified red fluorescent particles (excitation/emission = 520/580 nm) were dispersed in a solution. The contraction part was illuminated by light whose wavelength was 530 nm, from a mercury lamp (Olympus, Japan) which was filtered by XF102-2 (Omega optical, USA) through a 20× magnification objective lens. (NA = 0.4) Using a high-sensitive CCD camera (resolution 1000 × 1000 pixels) (Hamamatsu, Japan), the streak images of the light reflected from the fluorescent particles were captured at 30 frames per second [6]. The length of the upstream was 1 cm and that of downstream was 3 cm. Because the vortex in the upstream is known to be affected by the length of the downstream, the downstream length was designed to be three times the length of the upstream [16].

2.4. Flow dynamics

In this experiment, the following dimensionless numbers were defined in order to characterize the flow dynamics of the fluid inside the microchannel: the Weissenberg number (*Wi*), the Reynolds number (*Re*), and the Elasticity number (*El*) [6].

$$Wi = \lambda\bar{\dot{\gamma}}_c = \frac{\lambda V_c}{w_c/2} = \frac{\lambda Q}{hw_c^2/2} \tag{2.1}$$

$$\mathrm{Re} = \frac{\rho \bar{V}_c D_h}{\eta_0} = \frac{2\rho Q}{(w_c + h)\eta_0} \tag{2.2}$$

$$El = \frac{Wi}{Re} = \frac{2\lambda\eta}{\rho w_c D_h} = \frac{\lambda\eta(w_c + h)}{\rho w_c^2 h} \tag{2.3}$$

ρ is the fluid density; $\overline{V}_c$ is the average fluid velocity; w_c is the contraction width; *h* is the channel depth; η_0 is the zero-shear-rate viscosity; *Q* is the volumetric flow rate, and; D_h is the hydraulic diameter defined by $D_h = 2w_c h(w + h)$. λ is the relaxation time and $\bar{\dot{\gamma}}_c$ is the average shear-rate at the contraction throat.

Wi is defined as the characteristic time of the fluid multiplied by the shear rate at the contraction. This is the dimensionless deformation rate that the fluid experiences in the flow. *Wi* is often used for flows with a constant stretch history, and easily reaches large values due to the high shear rate in the microchannel.

Re is the ratio of the inertial force to the viscous force. In the microchannel flow, it is on the order of less than 10^{-1} and the inertia does not have a significant effect. These two dimensionless numbers are affected by the flow kinematics and increase with *Q*. The Elasticity number, *El*, is defined as the ratio of elastic force to the inertial force. As seen from the equation above, it is a dimensionless number that has no relationship with flow kinematics and depends only on the fluid properties and channel geometry. When various fluids and different microchannels are used as in this study, the El can be used to represent different experimental setups.

The experiment was first performed by letting the three PEO solutions (2 M 0.3 wt% (*El* = 90), 0.7wt% (*El* = 1900), 1.0 wt%