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# Nonlinear dynamic response of functionally graded shallow shells under harmonic excitation in thermal environment using finite element method

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ABSTRACT

The nonlinear dynamic response of functionally graded material (FGM) shallow shells subjected to thermal and harmonic loads is studied using finite element method. The material properties vary continuously in the thickness direction based on a simple power law distribution. The equations of motion are obtained using modal reduction method based on the third order shear deformation theory. The shooting method is used to obtain appropriate initial conditions for having only steady state response. The effects of thickness ratio and radii of curvature on the dynamic response of FGM shallow shells are studied. Buckled equilibrium positions (BEPs) are obtained for thermally loaded FGM shallow shells with immovable middle or physical neutral surfaces. It is shown that there exist one stable and two unstable BEPs. Then the behavior of an FGM shallow shell subjected to harmonic excitation in thermal environment is investigated. It is seen that the thermally loaded FGM shallow shell has different responses for the same frequency and amplitude of excitation depending on its initial conditions.


## 1. Introduction

Plate and shallow shells are common components in engineering structures. In many monographs various theories of shells have been established. These theories can be categorized in three groups: classical, first, and higher order shear deformation theories. The works of Donnell [1], Novozhilov [2], Sanders [3], Koiter [4], Ginsberg [5] are in the context of classical theories. In the first order shear deformation theory (FST), a uniform shear strain is assumed through the shell thickness. Therefore, a shear correction factor is required to take care of the non-uniformity in the shear stress distribution across the section. Higher order shear deformation theories (HST) express more accurate shear stress distribution through the shell thickness with respect to the FST. These theories have been completely discussed in the books of Amabili [6], Reddy [7] and Carrera et al. [8]. The validity and range of applicability of these theories were examined by Liew et al. [9]. Many researchers used these theories to study the nonlinear free and forced vibrations of FGM shallow shells. In the context of nonlinear free vibration of FGM shells, Liew et al. [10] studied the effects of various parameters on the nonlinear free vibration of three-layer coating-FGM-substrate cylindrical panel with different boundary conditions in the thermal environment. Chorfi and Houmat [11] used a p-version finite element method to study the nonlinear free vibration of FGM shallow shells of elliptical plan-form. In the context of nonlinear forced vibration of FGM shallow shells, Alijani and his co-workers [12–14] have done a comprehensive study. They used admissible functions to approximate the transverse deflection, rotation, and in-plane displacements. Only a few terms were used in the expansion of transverse deflection and rotation, while many terms were required to be used in the expansion of in-plane displacements for higher accuracy and better convergence. In reference [12], they studied chaotic vibrations of FGM shallow shells with internal resonance. Primary and sub-harmonic resonances of simply supported FGM shallow shells were studied by Alijani and Amabili [13] using multiple scale method. In reference [14], they studied the effect of thermal environment on the nonlinear vibration of FGM shallow shells based on the HST. Pradyumna and Nanda [15] studied nonlinear transient response of FGM shallow shells under suddenly applied load using higher-order finite element formulation.

In this study, for the first time, a nonlinear finite element formulation is proposed to study the dynamic response of FGM shallow shells under harmonic excitation in thermal environment.

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**Table 1**
Convergence study as a function of the number of elements for fundamental frequency parameter, $\hat{\omega} = \omega_{11} h \sqrt{\rho_c / E_c}$ for an Al/Al$_2$O$_3$ FGM shallow shell with simply supported edges movable in the middle surface ($a/b$ = 1, $h/a$ = 0.1).

| Geometry | $b/R_y$ | $a/R_x$ | $n$ | Number of modified MIN3 elements | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | 200 | 400 | 1152 | 1568 | 2048 |
| Spherical | 0.5 | 0.5 | 0.5 | 0.059320 | 0.059146 | 0.059128 | 0.059120 | 0.059110 |
| | | | 4 | 0.050044 | 0.049895 | 0.049844 | 0.049878 | 0.049861 |
| Hyperbolic parabolical | −0.5 | 0.5 | 0.5 | 0.048150 | 0.047658 | 0.047607 | 0.047575 | 0.047555 |
| | | | 4 | 0.037686 | 0.037315 | 0.036628 | 0.037265 | 0.037231 |

**Table 2**
Fundamental frequency parameter, $\hat{\omega} = \omega_{11} h \sqrt{\rho_c / E_c}$ for a simply supported Al/Al$_2$O$_3$ FGM shallow shell with movable edges ($a/b$ = 1, $h/a$ = 0.1).

| | $b/R_y$ | $a/R_x$ | $n$ | Present | Alijani et al. [13] | Chorfi and Houmat [11] | Matsunaga [21] |
|---|---|---|---|---|---|---|---|
| Plate | 0 | 0 | 0 | 0.0577 | 0.0597 | 0.0577 | 0.0588 |
| | | | 0.5 | 0.0491 | 0.0506 | 0.049 | 0.0492 |
| | | | 1 | 0.0442 | 0.0456 | 0.0442 | 0.043 |
| | | | 4 | 0.0381 | 0.0396 | 0.0383 | 0.0381 |
| | | | 10 | 0.0364 | 0.038 | 0.0366 | 0.0364 |
| Spherical | 0.5 | 0.5 | 0 | 0.0759 | 0.0779 | 0.0762 | 0.0751 |
| | | | 0.5 | 0.066 | 0.0676 | 0.0664 | 0.0657 |
| | | | 1 | 0.0602 | 0.0617 | 0.0607 | 0.0601 |
| | | | 4 | 0.049 | 0.0519 | 0.0509 | 0.0503 |
| | | | 10 | 0.0465 | 0.0482 | 0.0471 | 0.0464 |
| Cylindrical | 0 | 0.5 | 0 | 0.0626 | 0.0648 | 0.0629 | 0.0622 |
| | | | 0.5 | 0.0537 | 0.0553 | 0.054 | 0.0535 |
| | | | 1 | 0.0486 | 0.0501 | 0.049 | 0.0489 |
| | | | 4 | 0.0414 | 0.043 | 0.0419 | 0.0413 |
| | | | 10 | 0.0391 | 0.0408 | 0.0395 | 0.039 |
| Hyperbolic paraboloidal | −0.5 | 0.5 | 0 | 0.0572 | 0.0597 | 0.058 | 0.0563 |
| | | | 0.5 | 0.0487 | 0.0506 | 0.0493 | 0.0479 |
| | | | 1 | 0.0438 | 0.0456 | 0.0445 | 0.0432 |
| | | | 4 | 0.0378 | 0.0396 | 0.0385 | 0.0372 |
| | | | 10 | 0.0361 | 0.038 | 0.0368 | 0.0355 |

The equations of motion are obtained based on the HST. The order of equations of motion is reduced using modal reduction method. The effects of radii of curvature and thickness ratio on the nonlinear forced vibration of FGM shallow shells are studied. Moreover, the post-buckling behavior of FGM shallow shells under thermal load is investigated. It is shown that there are three BEPs for the thermally loaded FGM shallow shells with immovable simply supported edges (in middle surface or physical neutral surface). For an FGM shallow shell subjected to a harmonic excitation in thermal environment, the shooting method is used to obtain appropriate initial conditions for having purely steady state periodic response.

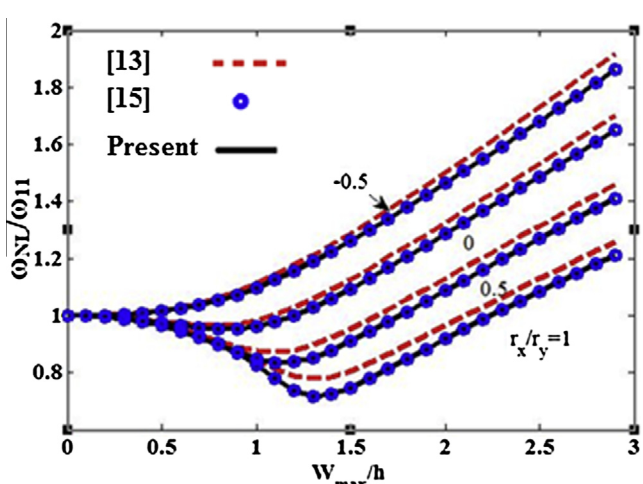


**Fig. 1.** Nonlinear free vibration of isotropic simply supported shallow shell with movable edges $r_x = \frac{R_x}{a}$, $r_y = \frac{R_y}{b}$, $E$ = 70 GPa, $\upsilon$ = 0.3, and $\rho$ = 2702 kg/m$^3$.

## 2. Formulation

### 2.1. The equations of motion in structural node DOF

Displacements at each point of an FGM shallow shell based on the higher order shear deformation theory can be presented as, [16]

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\left[\psi_y(x,y,t) - \frac{4}{3}\left(\frac{z}{h}\right)^2(\psi_y(x,y,t) + \varphi_y)\right] \\ v(x,y,z,t) &= v_0(x,y,t) + z\left[\psi_x(x,y,t) - \frac{4}{3}\left(\frac{z}{h}\right)^2(\psi_x(x,y,t) + \varphi_x)\right] \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \quad (1)$$

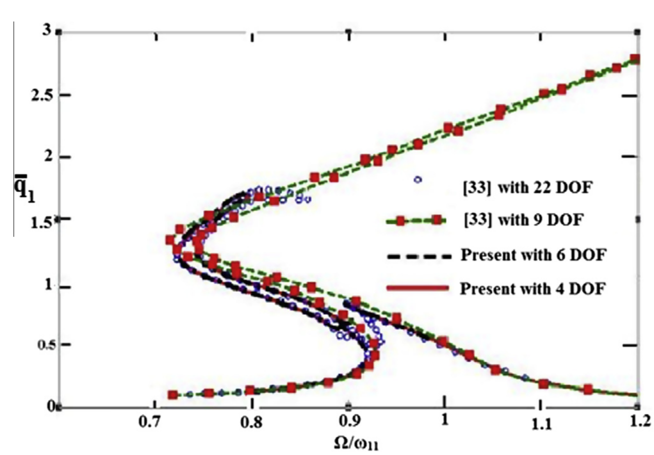


**Fig. 2.** MCM$_1$ of shell of simply supported movable edges $a = b$ = 0.1 m, $h$ = 0.001 m, $R_x = R_y$ = 1 m, $E$ = 206 GPa, $\upsilon$ = 0.3, $\rho$ = 7800 kg/m$^3$, $\hat{f}$ = 31.2 N, and $\zeta$ = 0.004.