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Simulations of an elastic particle in Newtonian and viscoelastic fluids subjected to confined shear flow

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ABSTRACT

The deformation and cross-streamline migration of an initially spherical neo-Hookean elastic particle suspended in confined shear flow of Newtonian and Giesekus viscoelastic fluids is studied through 3D arbitrary Lagrangian Eulerian finite element method numerical simulations. In both a Newtonian and a Giesekus liquid, when suspended in a symmetric position with respect to the walls of the flow cell, the particle deforms until reaching a steady ellipsoid-like shape, with a fixed orientation with respect to the flow direction. The dependences of such deformation and orientation on the flow strength, the geometric confinement, and the rheological properties of the suspending liquid are investigated. If the particle is initially closer to a wall of the channel than to the other, it also migrates transversally to the flow direction. In a Newtonian liquid, migration is always towards the center plane of the channel. In a Giesekus viscoelastic liquid, the migration direction depends on the competition between the elastic and the viscous forces arising in the suspending fluid; in a certain range of constitutive parameters, an 'equilibrium vertical position' in between the mid plane and the (upper/lower) wall of the channel is found, which acts as an attractor for particle migration.

1. Introduction

Elastic particles in a suspending liquid are systems of interest from both a scientific and a technological point of view: indeed, they can be regarded as models for more complicated systems, e.g. cells in biological flows, and can also be found as such in processing, e.g. filled polymers.

It is rather surprising that a somewhat limited attention has been devoted to such systems in the research literature; for sure, a wide comprehension of their mechanical behavior in flow is still lacking. In 1946, Fröhlich and Sack [1] investigated a suspension of elastic spheres in a Newtonian fluid under extensional flow, and derived an expression for the extensional stress of the suspension as a function of the strain rate. In 1967, Roscoe [2] studied theoretically the behavior of a dilute suspension of (visco)elastic spheres in a Newtonian fluid subjected to shear flow, predicting that the deformed particles attain a steady state, where they show an ellipsoidal shape with a fixed orientation with respect to the flow; the author also gave quantitative predictions of the deformation of the suspended particles, the stress, and the viscosity of the suspension as a function of the flow conditions and the constitutive properties of the particles and the suspending fluid. In the same year, Goddard and Miller [3] derived a constitutive equation for dilute suspensions of slightly deformed elastic spheres. In 1981, Murata [4] studied the small deformation of an initially spherical elastic particle in an arbitrary weak flow of a Newtonian fluid by means of a perturbative analysis. Since then, very little has been done on elastic particles until the end of the last decade, when Gao and Hu [5] performed 2D arbitrary Lagrangian Eulerian finite element method (ALE FEM) simulations. In 2011, the same group [6] studied the behavior of an initially spherical elastic particle suspended in a Newtonian fluid in shear flow through a non-perturbative method [7,8], coming to a validation and an extension of Roscoe's results. Very recently, Villone et al. [9] studied through 3D ALE FEM numerical simulations the behavior of an initially spherical elastic particle suspended in Newtonian and viscoelastic fluids under unbounded shear flow, validating the results in [2,6] and studying the effect of matrix elasticity on the dynamics and the steady state of an elastic particle, in terms of deformation and orientation.

In the present paper, the behavior of an initially spherical elastic particle suspended in a *confined shear flow* of a *Newtonian* and a

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viscoelastic liquid is studied by means of 3D ALE FEM numerical simulations. Due to the applied flow, the particle deforms; in addition, the presence of solid walls in its vicinity can make it migrate transversally to the streamlines of the suspending medium. Here, the effects of the geometrical and physical parameters of the system on both the deformation and the migration of the soft particle in both the Newtonian and the viscoelastic matrix are investigated.

The paper is organized as follows: in Section 2, the scheme of the problem and the governing equations are presented; in Section 3, some hints on the adopted numerical technique are given (details being available in [9]); in Section 4, results are presented; finally, in Section 5, some conclusions are drawn.

2. Mathematical model

In Fig. 1, a schematic drawing is reported of an initially spherical elastic particle suspended in a fluid under simple shear flow. For both the soft particle and the suspending phase, it is assumed that inertia can be neglected and that the volume is constant (i.e., the materials are incompressible). Therefore, the mass and momentum balance for both phases reduce to

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\nabla \cdot \boldsymbol{\sigma} = \boldsymbol{0} \tag{2}$$

where $\boldsymbol{u}$ and $\boldsymbol{\sigma}$ are the velocity vector and the stress tensor, respectively. $\boldsymbol{\sigma}$ can in turn be expressed as:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{T} \tag{3}$$

where p is the pressure, $\boldsymbol{I}$ is the identity tensor and $\boldsymbol{T}$ is the extra stress tensor.

For the extra-stress tensor $\boldsymbol{T}$, a constitutive equation has to be specified. For a Newtonian matrix, we have

$$\boldsymbol{T} = 2\eta_m \boldsymbol{D} \tag{4}$$

with η_m the viscosity, and $\boldsymbol{D}$ the symmetric part of the velocity gradient tensor ($\boldsymbol{D} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$). For a viscoelastic matrix fluid we write

$$\boldsymbol{T} = 2\mu_m \boldsymbol{D} + \boldsymbol{\tau} \tag{5}$$

with μ_m a viscosity and $\boldsymbol{\tau}$ the viscoelastic contribution to the extra-stress. For $\boldsymbol{\tau}$ we adopt the Giesekus (Gsk) model [10], which is given by

$$\lambda_m \overset{\triangledown}{\boldsymbol{\tau}} + \boldsymbol{\tau} + \frac{\alpha_m}{G_m}\boldsymbol{\tau}^2 = 2G_m\lambda_m \boldsymbol{D} \tag{6}$$

with λ_m the relaxation time, G_m the modulus, and α_m the so called 'mobility parameter'. The upper-convected derivative is defined by

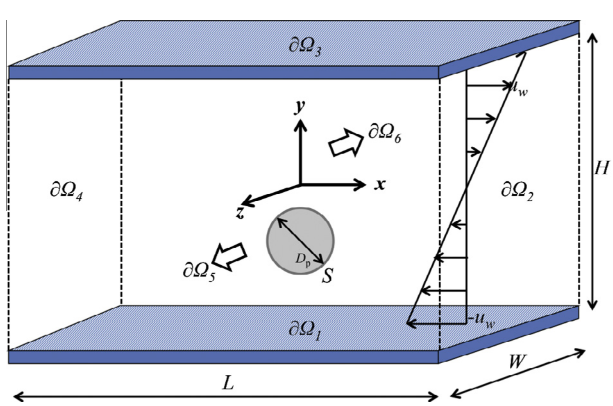

Fig. 1. Geometry of an initially spherical elastic particle suspended in a fluid under shear flow.

$$\overset{\triangledown}{\boldsymbol{\tau}} = \dot{\boldsymbol{\tau}} - (\nabla \boldsymbol{u})^T \cdot \boldsymbol{\tau} - \boldsymbol{\tau} \cdot \nabla \boldsymbol{u} \tag{7}$$

The mobility parameter α_m modulates the shear thinning of the Giesekus fluid, and it also induces a non-zero second normal stress difference N_2. For $\alpha_m = 0$, the Gsk model reduces to the Oldroyd-B model, which has a constant viscosity of

$$\eta_m = \mu_m + G_m\lambda_m \tag{8}$$

(We use in any event the same symbol η_m for the viscosity of the matrix, be it Newtonian or viscoelastic). For the general Gsk model, the viscosity becomes equal to η_m only in the limit of zero shear rate. If it is $\mu_m = 0$, the Gsk model reduces to the Upper Convected Maxwell (UCM) model, whose constitutive equation reads

$$\lambda_m \overset{\triangledown}{\boldsymbol{\tau}} + \boldsymbol{\tau} = 2G_m\lambda_m \boldsymbol{D} \tag{9}$$

Let us now introduce the constitutive equation of the particle. Even if it is quite common to use a displacement-based formulation for solids, our aim here is to maintain a conforming mesh across the interface and a mesh updating scheme (ALE) avoiding big mesh distortion within a short time. So, we decide to apply a velocity-based approach also for the solid. To do this, we consider the solid particle as a drop of UCM viscoelastic fluid with infinite relaxation time $\lambda_p \to \infty$ (more mathematical details are given in [9]). Then, Eq. (9), with G_m replaced by G_p (and λ_m replaced by λ_p) simply becomes

$$\overset{\triangledown}{\boldsymbol{\tau}} = 2G_p \boldsymbol{D} \tag{10}$$

which is the neo-Hookean elastic model with a modulus G_p.

The balance equations that describe the system shown in Fig. 1 are solved with the following boundary conditions:

$$\boldsymbol{u} = (-u_w, 0, 0) \text{ on } \partial\Omega_1 \tag{11}$$

$$\boldsymbol{u} = (u_w, 0, 0) \text{ on } \partial\Omega_3 \tag{12}$$

$$\boldsymbol{u}|_{\partial\Omega_2} = \boldsymbol{u}|_{\partial\Omega_4} \tag{13}$$

$$\boldsymbol{t}|_{\partial\Omega_2} = -\boldsymbol{t}|_{\partial\Omega_4} \tag{14}$$

$$\boldsymbol{u}|_{\partial\Omega_5} = \boldsymbol{u}|_{\partial\Omega_6} \tag{15}$$

$$\boldsymbol{t}|_{\partial\Omega_5} = -\boldsymbol{t}|_{\partial\Omega_6} \tag{16}$$

Eqs. (11) and (12) are the adherence conditions on the matrix velocity on the lower and the upper walls of the flow cell, respectively; Eqs. (13) and (14) express the periodicity of velocity and stress in the matrix along the flow direction, where the traction $\boldsymbol{t}$ is defined as: $\boldsymbol{t} = \boldsymbol{\sigma} \cdot \boldsymbol{m}$, with $\boldsymbol{m}$ the outwardly directed unit vector normal to the boundary; finally, Eqs. (15) and (16) are the periodical conditions on velocity and stress in the matrix along the vorticity direction.

The boundary conditions on the particle–matrix interface S are:

$$\boldsymbol{u}|_m = \boldsymbol{u}|_p \tag{17}$$

and

$$(\boldsymbol{\sigma}|_m - \boldsymbol{\sigma}|_p) \cdot \boldsymbol{n} = \boldsymbol{0} \tag{18}$$

where $\boldsymbol{n}$ is the outwardly directed unit vector normal to the interface. As an elastic solid is considered, no interfacial tension exists between the suspended particle and the suspending fluid; if a viscoelastic particle were investigated, on the contrary, the boundary condition on the interface would include a term accounting for interfacial tension (see, for example, Eq. (19) in [9]).

Since both the particle and the suspending medium are inertialess, no initial conditions on the velocities are required, whereas an initial condition is needed on the extra-stress tensor. We assume that the particle (and eventually the matrix, if it is viscoelastic) is initially stress-free, which means:

$$\boldsymbol{\tau}|_{t=0} = \boldsymbol{0} \tag{19}$$