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Falling film on flexible wall in the limit of weak viscoelasticity

Jie Peng [a], Yang-Jun Zhang [b,*], Wei-Lin Zhuge [b]

[a] Department of Engineering Mechanics, Tsinghua University, PR China
[b] State Key Laboratory of Automotive Safety and Energy, Tsinghua University, Beijing 100084, PR China

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ABSTRACT

The flow dynamics of an upper-convected-Maxwell (UCM) falling film down a flexible vertical wall is studied in the limit of weak viscoelasticity. A set of Benney-like weakly nonlinear equations for the film thickness and wall deflection, which is valid for small flow rate, is derived based on the long-wave theory. It shows that the unstable role of liquid viscoelasticity is equivalent to that of the flow inertia. A set of asymptotic evolution equations valid for moderate flow rate is obtained based on the integral theory. The linear instability property of the system is examined by using a normal-mode analysis. It shows that the liquid viscoelasticity acts to destabilize the falling film even for the flow with inertia being negligible. The nonlinear evolution equations for the moderate flow rate are solved numerically. The spatio-temporal evolutions of the liquid–air interface and flexible wall are examined. It is concluded that the liquid viscoelasticity plays a role to strengthen the dispersion of the initial imposed perturbation. It can promote the traveling speed of the solitary-like humps and suppress the front-running ripples at the same time. Both the wall damping and wall tension acts to suppress the fluctuations of the flexible wall. However, they play different roles in the evolution of the liquid–air interface.

1. Introduction

Falling film occurs in various technical processes, environmental sciences and everyday life [1–3]. It has been studied over past decades both experimentally and theoretically. Much of this work has been summarized in a book [4]. For the flow with small Reynolds number, it is feasible to develop a long-wave weakly nonlinear equation like Benney [5] or Kuramoto–Sivashinsky [6] equation. For the flow with moderate Reynolds number, a coupled system of evolution equations for the film thickness and volumetric flow rate can be derived alternatively [7]. Khayat [8] studied the influence of substrate topography on the transient flow field of the coating film. It was concluded that the topography of the substrate has a drastic effect on the flow of film. The studies cited in the brief review above have all considered the liquid film flowing over a rigid wall. In recent years, the dynamics of falling film over flexible walls are of interest since it occurs in a wide range of situations. These include the modeling of airflow in pulmonary airways [9,10], the use of rubber-covered rolls to reduce defects in a coating process [11,12], and so on. It is expected that the wall flexibility can have a considerable effect on dynamics of the falling film [13]. Matar et al. [14] considered the dynamic behavior of falling Newtonian liquid-film over a flexible wall. Based on the long-wave theory, a set of coupled equations for the film thickness and substrate deflection was derived with a small flow Reynolds number. In addition, a set of equations coupled for the film thickness, substrate deflection and film volumetric flow rate was also presented, which is valid for moderate Reynolds numbers. It was pointed out that decreasing the wall damping and/or wall tension can promote the development of chaos or severe substrate deformation. Sisoev et al. [15] revisited this problem with evolution equations being re-derived through the Shkadov approach. The results can be reducible to those for the flow of a falling film on a rigid wall.

A vast majority of studies on thin-film flow problems were devoted to the flow of Newtonian fluid. The film flow of non-Newtonian fluid attracted less attention over the past. In recent decades, the flow of viscoelastic fluid, a subclass of non-Newtonian fluids, has emerged as a research subject of great interest. The viscoelastic nature of most polymeric fluids can give rise to new mechanisms, which can affect the flow instability caused by capillary or inertial forces [16]. From a purely fluid dynamical standpoint, the viscoelastic fluid exhibits a great deal of influence on the normal and shear stresses in flow film. It is expected that the liquid viscoelasticity can have a considerable effect on dynamics of falling film. On a more applied level, viscoelastic fluids are widely used in analysis to characterize the lubrication behaviors of bearings, gears and cams. Gupta [17] studied the instability

* Corresponding author.
E-mail address: yjzhang@tsinghua.edu.cn (Y.-J. Zhang).

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properties of a falling viscoelastic liquid film and concluded that the viscoelasticity can destabilize the film flow. Similar results were also obtained by Shaqfen et al. [18] for the film flow with small Reynolds number. However, for moderate Reynolds numbers, it was pointed out that the viscoelastic effects are primarily stabilizing. Andersson [19] considered the steady laminar film flow of viscoelastic Walters' liquid down a vertical wall. The resulting analytical expression for the film thickness reveals that the viscoelastic film grows up faster towards the downstream asymptotic traveling wave state than that of the Newtonian film. Khayat [20,21] studied the effect of substrate topography on the flow of Oldroyd-B film. It was concluded that the topography of the substrate has a drastic effect on film flow. Besides, they also claimed that the liquid elasticity had a significant effect on both the steady state and transient behavior of the system. Sergey [22] studied the viscoelastic flow over a step-down topography in the presence of inertia. It was pointed out that liquid viscoelasticity has a monotonically decreasing effect on the height of a capillary ridge. Pavlidis et al. [23] simulated the flow of the viscoelastic film, which was modeled with an exponential Phan-Thien and Tanner (ePTT) constitutive equation, over 2D topography by using a mixed finite-element method. Again, for the flow with weak viscoelasticity, it was concluded that the capillary ridge decreases with increasing of viscoelasticity. However, for the flow with relatively strong viscoelasticity, the situation is reversed because the shear and elongation thinning become more important.

In this study, we consider a viscoelastic free-surface liquid film flows over a flexible wall. It extends Matar et al. [14]'s analysis to include the effect of the liquid viscoelasticity. The upper-convected-Maxwell (UCM) constitutive equation is adopted to model the viscoelastic liquid. For flexible wall, the effects of wall damping and tensions are included with the wall inertia being neglected. This is the simplest system which couples a restoring force with the normal force imposed by the fluid, while the bending stresses are neglected. The effects of flexible wall and liquid viscoelasticity on the film dynamics are mainly concerned. The structure of the paper is as follows. In Section 2, we use the long-wave theory to derive a pair of coupled equations for film thickness and wall deflection, which is valid for small flow rate. For moderate flow rate, alternatively, a set of asymptotic evolution equations for the film thickness, volumetric flow rate and the wall deflection is obtained by using the integral theory. In both cases, the linear instability analysis is presented in Section 3. The numerical solutions to the nonlinear system are presented in Section 4, and some concluding remarks are given in Section 5.

2. Formulation

2.1. Governing equations

We consider a two-dimensional incompressible viscoelastic UCM liquid film flow down an infinitely long vertical flexible wall under the effect of gravity force, as illustrated in Fig. 1. Cartesian coordinate system (x, y) is introduced with the x axis oriented downwards along the non-perturbed wall. The y axis denotes the normal direction. The liquid film with density ρ and viscosity μ occupies the region $\zeta(x,t) \leqslant y \leqslant \eta(x,t)$ and is bounded by an inviscid gas. $\eta(x,t)$ and $\zeta(x,t)$ denote the position of liquid–air interface and flexible wall, respectively. Thickness of the undisturbed film is assumed to be h_0. Flow in the liquid film can be modeled by using continuity and Navier–Stokes equations, which are given below

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

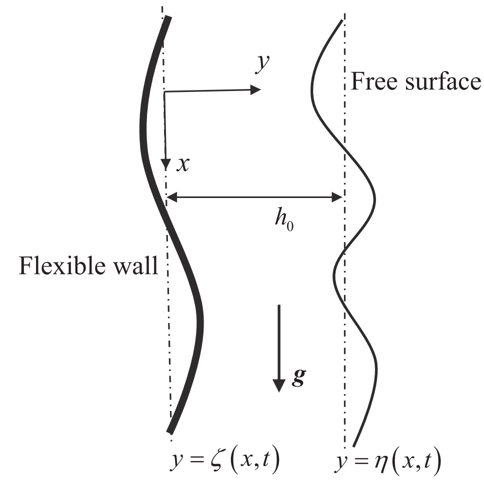

Fig. 1. Schematic of the problem.

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \rho g, \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}. \tag{3}$$

Here, p and ρ denote the pressure and density, g is the gravitational acceleration, which is assumed to be along x direction, u and v represent the flow velocity in x and y direction, respectively. $\sigma_{xx}, \sigma_{xy}, \sigma_{yx}$ and σ_{yy} denote the component of deviatoric stress tensor with σ_{yx} equaling to σ_{xy}. For UCM fluid, we have the following expressions [24]

$$\sigma_{yy} + \lambda\left(\frac{\partial \sigma_{yy}}{\partial t} + u\frac{\partial \sigma_{yy}}{\partial x} + v\frac{\partial \sigma_{yy}}{\partial y}\right) - 2\lambda\left(\sigma_{yy}\frac{\partial v}{\partial y} + \sigma_{yx}\frac{\partial v}{\partial x}\right) = 2\mu\frac{\partial v}{\partial y}, \tag{4}$$

$$\sigma_{xx} + \lambda\left(\frac{\partial \sigma_{xx}}{\partial t} + u\frac{\partial \sigma_{xx}}{\partial x} + v\frac{\partial \sigma_{xx}}{\partial y}\right) - 2\lambda\left(\sigma_{xy}\frac{\partial u}{\partial y} + \sigma_{xx}\frac{\partial u}{\partial x}\right) = 2\mu\frac{\partial u}{\partial x}, \tag{5}$$

$$\sigma_{yx} + \lambda\left(\frac{\partial \sigma_{yx}}{\partial t} + u\frac{\partial \sigma_{yx}}{\partial x} + v\frac{\partial \sigma_{yx}}{\partial y}\right) - \lambda\left(\sigma_{xx}\frac{\partial v}{\partial x} + \sigma_{yy}\frac{\partial u}{\partial y}\right) = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \tag{6}$$

where λ is the relaxation time and μ is the zero-shear-rate viscosity. The flexible wall is assumed to be infinitely long. The wall deflection along x direction is ignored since the wall is assumed to be tethered and only the long-wavelength perturbations are considered in this study. Corresponding dynamics are governed by a forced membrane equation [3,9]

$$\frac{\rho_w H_w \alpha}{\left(1+\zeta_x^2\right)^{1/2}}\frac{\partial \zeta}{\partial t} - \frac{T_w}{\left(1+\zeta_x^2\right)^{3/2}}\frac{\partial^2 \zeta}{\partial x^2} = -p + p_w + \frac{\sigma_{yy} + \zeta_x^2\sigma_{xx}}{1+\zeta_x^2} - \frac{2\zeta_x\sigma_{xy}}{1+\zeta_x^2}. \tag{7}$$

Here, the definition of ζ_x is $\zeta_x = \partial\zeta/\partial x$. ρ_w , H_w and α denote the density, thickness, and damping coefficient of the flexible wall. T_w is the wall tension, which remains uniformly across the thickness. p_w represents the pressure external to the wall and is assumed to be zero without loss of generality.