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An intuitive computational multi-scale methodology and tool for the dynamic modelling of viscoelastic composites and structures

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ABSTRACT

This paper proposes an intuitive computational multi-scale homogenisation procedure and tool for the estimation of the effective static and mechanical properties of complex viscoelastic composite material and structures. The proposed solution consists in computing numerically the complex effective properties (storage and loss moduli) as function of frequency. The developed numerical tool is coupled with ABAQUS FEA in order to ease the uptake of the technology code and allows for an accurate and automatic simulation of composite engineering structures with substantially less human intervention and a rational control of the error. Details concerning the numerical implementation and a selection of representative numerical examples are provided.

1. Introduction

Structural vibrations control is of primary importance for enhancing safety and improving structures and systems performances. The most prevalent strategy for structural vibration damping is the one that consider adding viscoelastic inhomogeneities either as thin layers or as dilute inclusions depending on the practical considerations. These composite materials and structures yield superior vibrational energy absorption capability. They particularly offer the advantage of high damping, low cost, ease of implementation with low weight.

Analytical studies were devoted to simple structures and finite element simulations were introduced to design structures with complex geometries and generic boundary conditions. Most analyses use a complex frequency-independent dynamic modulus to describe the rheological behaviour of viscoelastic materials. However, it is widely known from engineering practice, that the storage and loss moduli of viscoelastic materials are strongly frequency and temperature dependent. This dependence is especially significant for elastomers and polymers, such as those used in laminated composites. A comprehensive review of the various research methods and theory calculation models could be found in the very recent review by Zhou [1].

Nevertheless, viscoelasticity is not the only mechanism for damping. As a matter of fact, defects and interfaces can also contribute to damping vibrational dissipating energy. Therefore, the micro-structure greatly affects the damping ability of a material. Consequently, new materials with tailored micro-structure have attracted increased interest because of the possibility to control both their stiffness and damping characteristics i.e. loss modulus. In parallel, the recent advancements in micron-level additive manufacturing have enabled unprecedented accuracy in controlling the material architecture resulting in an unrestricted number of possibilities for multi-scale material design. In view of acceptance of these new composite materials for vibration damping, the development of models and numerical solutions and tools is obvious. It is through these models that the effect of shapes, sizes and location of phase inclusions on the dynamical properties can be investigated. These models should be capable of correlating the micro-structural response with the overall macroscopic dynamical behaviour since the applied loads are at the structural level.

A series of micro-mechanical approaches for the estimation of the damping properties of composite materials, as a function of the properties of the constituent materials have been developed in [2–12]. Generally, the effective behaviour of the viscoelastic composite in the time domain is obtained using a combination of the correspondence principle with the homogenization solution of the corresponding elastic problem. Then, the solution of the viscoelastic effective properties is obtained using the Laplace Carson transform technique. This permits transferring a viscoelastic

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problem into a symbolic elastic problem with known solution. The main technical issue when using these analytical methods towards up-scaling of viscoelastic properties of composites, is performing the inverse Laplace transform since the expressions of the effective properties in the Laplace space are not rational fractions as functions of the Laplace variable. Even sophisticated approaches have been developed and used, such as in [13,14], to perform the inverse transform, basically, all these analytical and semi analytical methods are restricted to canonical micro-structure shapes like sphere or cylinder like shapes. Recent works proposed the use finite element method (FEM) solution in multi-scale context where the macroscopic parameters are obtained by volume averaging over statistically representative volume which are then used at the macroscopic level [15–21]. This latter numerical framework can be an alternative approach to answer the previous drawbacks. Typically, this approach is based on a hierarchical decomposition of the solution space into a local solution and a global one and by enforcement of the solution compatibility conditions.

A multilevel finite element methodology is introduced where the hierarchical character of model description and simulation results are exploited to expedite the analysis of problems. This makes it ideal for local/global analyses where solutions from a local model, i.e. the micro-structure, are used to derive the solution for the global model of the structure. In this paper, an intuitive numerical procedure for the estimation of the effective properties of the viscoelastic composites is developed and a computational multilevel methodology and tool for the prediction of dynamical properties of composite structures is proposed as an alternative to the direct simulation which requires enormous computing resources. The proposed solution consists of computing numerically the complex effective proprieties (storage and loss moduli) as function of frequency. This is obtained using a steady state dynamic direct solver with a sweep over the frequency band of interest and then simulating the Mechanical Impedance (MI) [22] test for an equivalent homogeneous structure in order to predict the spectral response in terms of driving point mobility. When compared to the direct numerical simulation, this method ensures a considerable saving in computational effort. The method is suitable to deal with composites with complex micro-structure and to cope with the inaccuracy of traditional constitutive relations. It is also suitable and effective to simulate composites with high volume fraction. Hence, it allows for tailoring the effective properties to specific application requirement. The developed numerical tool is coupled with ABAQUS FEA in order to ease the uptake of the technology code and allow for automatic simulation of complex 3D composite engineering structures and boundary conditions. Details concerning the numerical implementation and a selection of representative numerical examples are provided.

2. Multi-level FEM approach and problem formulation

The multilevel principle assumes that if a heterogeneous structure (Ω) contains evenly distributed micro-topologies, a Representative Volume Element (RVE) ($\Omega_{RVE} \ll \Omega$) can be, then, defined where the averaged micro-structural behaviour can be smeared as an homogeneous one over the macroscopic domain (Ω) (see Fig. 1). A concise description of the particular technique of multilevel finite element method, that constitutes the first significant part of the approach described in the current work, can be found in [17,19,21].

2.1. Problem at the macro-scale

The problem at the macro-scale is governed by the equation of motion, which is an expression of Newton's second law:

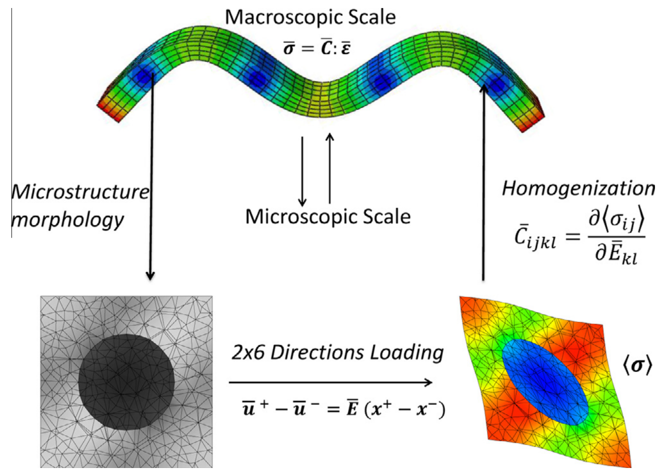

Fig. 1. Schematic representation of the multilevel multi-scale approach.

$$\rho\frac{\partial^2\bar{\boldsymbol{U}}(\boldsymbol{x},t)}{\partial t^2} + \nabla\cdot\bar{\boldsymbol{\sigma}}(\boldsymbol{x},t) = \bar{\boldsymbol{F}}(\boldsymbol{x},t) \tag{1}$$

where $\bar{\boldsymbol{U}} = [\bar{u}_1, \bar{u}_2, \bar{u}_3]$ is the displacement vector, the bar sign $(\bar{\cdot})$ denotes the macroscopic value, $\boldsymbol{x}$ is the vector position and t is the time. ρ stands for density. The composite material is considered as viscoelastic, thus the stress $\boldsymbol{\sigma}(\boldsymbol{x},t)$ is related to the strain $\boldsymbol{\epsilon}(\boldsymbol{x},t)$ by the Boltzmann convolution integral:

$$\bar{\boldsymbol{\sigma}}(\boldsymbol{x},t) = \int_0^t \bar{\boldsymbol{C}}(t-\tau) : \frac{\partial}{\partial\tau}\bar{\boldsymbol{\varepsilon}}(\boldsymbol{x},\tau)\mathrm{d}\tau \tag{2}$$

while t is the instant of loading, τ spans the history time and $\boldsymbol{C}$ is the relaxation modulus. If we apply the Laplace–Carson transformation, $\mathscr{L}^{\mathscr{C}}[f(t)] = f^*(p) = p\int_0^t f(t)e^{-pt}dt$, to the convolution integral in Eq. (2), the constitutive relation becomes a simple tensor contraction in the Laplace–Carson space:

$$\mathscr{L}^{\mathscr{C}}[\bar{\boldsymbol{\sigma}}(\boldsymbol{x},t)] = \bar{\boldsymbol{\sigma}}^*(\boldsymbol{x},p) = \bar{\boldsymbol{C}}^*(p) : \bar{\boldsymbol{\varepsilon}}^*(\boldsymbol{x},p) \tag{3}$$

with $p = \alpha + \mathrm{i}\omega$ and $\mathrm{i} = \sqrt{-1}$. α is an arbitrary real value and the Laplace–Carson domain is, then, the frequency domain. $\bar{\boldsymbol{C}}^*(p) \equiv \bar{\boldsymbol{C}}(\omega)$ is the complex fourth order tangent modulus tensor:

$$\bar{\boldsymbol{C}}(\omega) = \bar{\boldsymbol{C}}'(\omega) + \mathrm{i}\bar{\boldsymbol{C}}''(\omega) \tag{4}$$

where ω is the angular frequency. By considering the excitation force to be a monochromatic plane wave, the displacement field in the medium is also a monochromatic plane wave:

$$\bar{\boldsymbol{U}}(\boldsymbol{x},t) = \bar{\boldsymbol{U}}(\boldsymbol{x})\exp(-\mathrm{i}\omega t) \tag{5}$$

By replacing the displacement form in Eq. (5) and the constitutive law in Eq. (4) in the governing equation, Eq. (1), the time-free wave equation of elasto-dynamics is obtained:

$$\rho\omega^2\bar{u}_i + \partial_j\bar{C}_{ijkl}\partial_l\bar{u}_k = \bar{f}_i \tag{6}$$

This system can be solved using a numerical perturbation procedure, where the perturbed solution is obtained by a linearisation in the current base state. Structural and viscous damping can be included in the numerical procedure using the Rayleigh and structural damping coefficients. Besides, global damping coefficients can be specified at the procedure level to define additional viscous and structural damping contributions. In the context of a multilevel finite element analysis, the stress field at the macro-scale level can be computed by solving a local nonlinear finite element problem.