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Nonlinear two-scale shell modeling of sandwiches with a comb-like core

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ABSTRACT

In this paper a computational two-scale model for sandwich composites with a comb-like core structure is proposed. On the global scale the sandwich panel is modeled with homogenized finite shell elements, while on the local scale a representative volume element (RVE) describes the microstructure of the sandwich through the full thickness. The local model is implemented as a constitutive law for the shell elements on the global scale. As the material response is updated in each iteration step of a nonlinear simulation, local effects of nonlinearity such as face sheet buckling or plastic flow can be described by the model.

1. Introduction

The growing prevalence of modern composite materials and lightweight construction elements in practical applications of various engineering disciplines creates a demand for calculation methods which can accurately describe the mechanical behavior of a composite structure, while at the same time preserving moderate requirements in terms of numerical cost. Classical homogenization methods have been developed as a remedy to this issue, in which the complex microstructure is replaced by a homogeneous material using averaged mechanical properties that are determined via experiments [1] or by analytical [2] and/or numerical [3,4] investigation. Some of these methods are described in review articles concerned with the modeling of sandwich structures such as [5,6]. The concept of multiscale methods as a computational homogenization scheme is by now well established in the literature, e.g. [7–9]. However, many of the presented methods are unable to reflect nonlinear system behavior on the local scale, where the effective properties, which are presumed to be constant, begin to change over the course of a simulation. While many experimental works, e.g. [1,10], establish failure mode estimates, the nonlinear deformation behavior such as in the post-critical state can not be described.

In this paper a coupled global–local method will be presented specifically for sandwich panels with axially stiffened or honeycomb cores, in extension to an earlier approach for layered structures [11]. For this means a global model of the whole structure is discretized with quadrilateral shell elements [12] and then coupled with multiple local models, describing the sandwich microstructure through the full thickness and using shell elements for discretization as well. The local scale model is implemented as a constitutive law for the global model, so that one local boundary value problem is evaluated in each integration point of the global structure. They are reevaluated in each iteration step in a nonlinear simulation so that physical and geometrical nonlinearity can be described. It will be shown in numerical examples that elasto-plastic material behavior and pre- and post-critical buckling behavior are reflected correctly, contrary to most classical homogenization methods. A more comprehensive work covering the method presented here, next to additional numerical examples and further side aspects, is given in [13].

2. Formulation of the two-scale boundary value problem

Let $\mathcal{B}_0$ be the domain of a shell with thickness h in the Euclidean space in the reference configuration. The reference surface Ω_0 with boundary Γ_0 is introduced. Accordingly, $\mathcal{B}_t, \Omega_t$ and Γ_t denote the shell space, reference surface and boundary in the current configuration. In the following, Latin indexes $i, j, \ldots$ go from 1 to 3, Greek indexes α, β from 1 to 2 and the summation convention is employed. A convective coordinate system is introduced with ξ^i. The expression $(.)_{,i}$ denotes the partial derivative of a quantity $(.)$ with respect to the coordinate ξ^i. ξ^3 represents the thickness direction so that $\xi^3 \in [h^-, h^+]$ is given. Points in the shell space are identified with a position vector $\mathbf{X}$ to a point in the reference surface and a director vector $\mathbf{D}$, which is perpendicular to the reference surface in the reference configuration:

$$\mathbf{\Phi}(\xi^1, \xi^2, \xi^3) = \mathbf{X}(\xi^1, \xi^2) + \xi^3 \mathbf{D}(\xi^1, \xi^2) \tag{1}$$

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In the same way, points in the deformed shell space of the current configuration are identified with a position vector $\mathbf{x}$ to the deformed reference surface and the corresponding director $\mathbf{d}$:

$$\boldsymbol{\phi}(\xi^1,\xi^2,\xi^3) = \mathbf{x}(\xi^1,\xi^2) + \xi^3\mathbf{d}(\xi^1,\xi^2) \tag{2}$$

Both director vectors are presumed to have the length 1, $|\mathbf{D}| \equiv |\mathbf{d}| \equiv 1$, enforcing an inextensibility condition. However, the director vector in the current configuration is not required to be perpendicular to the reference surface Ω_t, in order to allow for transverse shear strains within the context of the Reissner–Mindlin theory. The displacement field $\mathbf{u}$ can then be introduced:

$$\mathbf{u}(\xi^1,\xi^2) = \mathbf{x}(\xi^1,\xi^2) - \mathbf{X}(\xi^1,\xi^2) \tag{3}$$

The shell strains

$$\boldsymbol{\varepsilon} = [\varepsilon_{11},\varepsilon_{22},2\varepsilon_{12},\kappa_{11},\kappa_{22},2\kappa_{12},\gamma_1,\gamma_2]^T \tag{4}$$

are obtained based on the Green–Lagrange strain tensor with components

$$\begin{aligned}
\varepsilon_{\alpha\beta} &= \frac{1}{2}\left(\mathbf{x}_{,\alpha}\cdot\mathbf{x}_{,\beta} - \mathbf{X}_{,\alpha}\cdot\mathbf{X}_{,\beta}\right)\\
\kappa_{\alpha\beta} &= \frac{1}{2}\left(\mathbf{x}_{,\alpha}\cdot\mathbf{d}_{,\beta} + \mathbf{x}_{,\beta}\cdot\mathbf{d}_{,\alpha} - \mathbf{X}_{,\alpha}\cdot\mathbf{D}_{,\beta} - \mathbf{X}_{,\beta}\cdot\mathbf{D}_{,\alpha}\right)\\
\gamma_\alpha &= \mathbf{x}_{,\alpha}\cdot\mathbf{d} - \mathbf{X}_{,\alpha}\cdot\mathbf{D}
\end{aligned} \tag{5}$$

The second-order curvatures $\rho_{\alpha\beta}$ are neglected for a thin shell. Normal strains in thickness direction do not occur due to the inextensibility assumption. The vectors of stress resultants and couple stress resultants $\mathbf{n}^\alpha$ and $\mathbf{m}^\alpha$ are introduced as integrals of the first Piola–Kirchhoff stress tensor $\mathbf{P}$ over the shell thickness:

$$\begin{aligned}
\mathbf{n}^\alpha &= \int_{h^-}^{h^+} \mathbf{P}\mathbf{G}^\alpha\,\bar{\mu}\,\mathrm{d}\xi^3\\
\mathbf{m}^\alpha &= \mathbf{d}\times\int_{h^-}^{h^+} \mathbf{P}\mathbf{G}^\alpha\,\xi^3\,\bar{\mu}\,\mathrm{d}\xi^3
\end{aligned} \tag{6}$$

Here, $\mathbf{G}^\alpha$ are the contravariant base vectors and $\bar{\mu}$ is defined with the help of the volume element $\mathrm{d}V = \bar{\mu}\,\mathrm{d}\xi^3\,\mathrm{d}A$ and the area element $\mathrm{d}A = j\,\mathrm{d}\xi^1\,\mathrm{d}\xi^2$ of the reference surface with $j = |\mathbf{X}_{,1}\times\mathbf{X}_{,2}|$. Decomposition of the stress resultants $\mathbf{n}^\alpha$ and director stress resultants $\mathbf{m}^\alpha$ into their components along $\mathbf{x}_{,\alpha}$ and $\mathbf{d}$,

$$\begin{aligned}
\mathbf{n}^\alpha &= n^{\alpha\beta}\mathbf{x}_{,\beta} + q^\alpha\mathbf{d} + m^{\alpha\beta}\mathbf{d}_{,\beta}\\
\mathbf{m}^\alpha &= \mathbf{d}\times m^{\alpha\beta}\mathbf{x}_{,\beta}
\end{aligned} \tag{7}$$

leads to the membrane forces $n^{\alpha\beta} = n^{\beta\alpha}$, bending moments $m^{\alpha\beta} = m^{\beta\alpha}$ and shear forces q^α. With these, the vector of stress resultants $\boldsymbol{\sigma}$ is introduced:

$$\boldsymbol{\sigma} = \left[n^{11},n^{22},n^{12},m^{11},m^{22},m^{12},q^1,q^2\right]^T \tag{8}$$

In order to obtain the equilibrium conditions in the shell formulation, the variations of the shell strains $\delta\boldsymbol{\varepsilon}$ must be derived which leads to

$$\begin{aligned}
\delta\varepsilon_{\alpha\beta} &= \frac{1}{2}\left(\delta\mathbf{x}_{,\alpha}\cdot\mathbf{x}_{,\beta} + \delta\mathbf{x}_{,\beta}\cdot\mathbf{x}_{,\alpha}\right)\\
\delta\kappa_{\alpha\beta} &= \frac{1}{2}\left(\delta\mathbf{x}_{,\alpha}\cdot\mathbf{d}_{,\beta} + \delta\mathbf{x}_{,\beta}\cdot\mathbf{d}_{,\alpha} + \delta\mathbf{d}_{,\alpha}\cdot\mathbf{x}_{,\beta} + \delta\mathbf{d}_{,\beta}\cdot\mathbf{x}_{,\alpha}\right)\\
\delta\gamma_\alpha &= \delta\mathbf{x}_{,\alpha}\cdot\mathbf{d} + \delta\mathbf{d}\cdot\mathbf{x}_{,a}
\end{aligned} \tag{9}$$

They are arranged in the vector of virtual shell strains $\delta\boldsymbol{\varepsilon}$ conforming to (4),

$$\delta\boldsymbol{\varepsilon} = [\delta\varepsilon_{11},\delta\varepsilon_{22},2\delta\varepsilon_{12},\delta\kappa_{11},\delta\kappa_{22},2\delta\kappa_{12},\delta\gamma_1,\delta\gamma_2]^T \tag{10}$$

The internal virtual work can then be expressed in terms of the stress resultants via

$$\delta W_{\mathrm{int}} = \int_{\Omega_0} \delta\boldsymbol{\varepsilon}^T\boldsymbol{\sigma}\,\mathrm{d}A \tag{11}$$

and inserting the components of $\delta\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ as well as (7),

$$\delta W_{\mathrm{int}} = \int_{\Omega_0} \left(\mathbf{n}^\alpha\cdot\delta\mathbf{x}_{,\alpha} + q^\alpha\mathbf{x}_{,\alpha}\cdot\delta\mathbf{d} + m^{\alpha\beta}\mathbf{x}_{,\beta}\cdot\delta\mathbf{d}_{,\alpha}\right)\mathrm{d}A \tag{12}$$

is obtained. The boundary Γ_0 of the shell is separated into a part $\Gamma_{0\sigma}$ with stress boundary conditions and a part Γ_{0u} with displacement boundary conditions. Admissible variations must fulfill the kinematic boundary conditions $\delta\mathbf{x} = \delta\mathbf{u} = \delta\mathbf{d} = 0$ on Γ_{0u}. With (12) and integration by parts it follows

$$\begin{aligned}
\delta W_{\mathrm{int}} = &-\int_{\Omega_0}\left[\frac{1}{j}(j\mathbf{n}^\alpha)_{,\alpha}\cdot\delta\mathbf{u} + \left(\frac{1}{j}(j\mathbf{m}^\alpha)_{,\alpha} + \mathbf{x}_{,\alpha}\times\mathbf{n}^\alpha\right)\cdot\delta\boldsymbol{\omega}\right]\mathrm{d}A\\
&+\int_{\Gamma_{0\sigma}}\left[j(\mathbf{n}^\alpha\nu_\alpha)\cdot\delta\mathbf{u} + j(\mathbf{m}^\alpha\nu_\alpha)\cdot\delta\boldsymbol{\omega}\right]\mathrm{d}s,
\end{aligned} \tag{13}$$

with rotational parameters $\boldsymbol{\omega}$ and the outward normal vector ν_α on $\Gamma_{0\sigma}$. Due to the kinematic boundary conditions no contributions on Γ_{0u} occur. External surface loads $\bar{\mathbf{p}}$ on the reference surface Ω_0 and boundary loads $\bar{\mathbf{t}}$ on $\Gamma_{0\sigma}$ are incorporated in the external virtual work:

$$\delta W_{\mathrm{ext}} = \int_{\Omega_0}\bar{\mathbf{p}}\cdot\delta\mathbf{u}\,\mathrm{d}A + \int_{\Gamma_{0\sigma}}\bar{\mathbf{t}}\cdot\delta\mathbf{u}\,\mathrm{d}s \tag{14}$$

Consequently, the principle of virtual work for the shell can be obtained with $\mathbf{v} := [\mathbf{u},\boldsymbol{\omega}]^T$:

$$g(\mathbf{v},\delta\mathbf{v}) = \delta W_{\mathrm{int}} - \delta W_{\mathrm{ext}} = 0 \tag{15}$$

and inserting (13) and (14) leads to the representation

$$\begin{aligned}
g(\mathbf{v},\delta\mathbf{v}) = &-\int_{\Omega_0}\left[\left(\frac{1}{j}(j\mathbf{n}^\alpha)_{,\alpha} + \bar{\mathbf{p}}\right)\cdot\delta\mathbf{u} + \left(\frac{1}{j}(j\mathbf{m}^\alpha)_{,\alpha} + \mathbf{x}_{,\alpha}\times\mathbf{n}^\alpha\right)\cdot\delta\boldsymbol{\omega}\right]\mathrm{d}A\\
&+\int_{\Gamma_{0\sigma}}\left[(j\mathbf{n}^\alpha\nu_\alpha - \bar{\mathbf{t}})\cdot\delta\mathbf{u} + j(\mathbf{m}^\alpha\nu_\alpha)\cdot\delta\boldsymbol{\omega}\right]\mathrm{d}s.
\end{aligned} \tag{16}$$

The fundamental lemma of variational calculus is then applied and with arbitrarily chosen admissible variations $\delta\mathbf{u},\delta\boldsymbol{\omega}$ the static equilibrium conditions follow

$$\begin{aligned}
&\frac{1}{j}(j\mathbf{n}^\alpha)_{,\alpha} + \bar{\mathbf{p}} = 0\\
&\frac{1}{j}(j\mathbf{m}^\alpha)_{,\alpha} + \mathbf{x}_{,\alpha}\times\mathbf{n}^\alpha = 0 \quad \text{in } \Omega_0
\end{aligned} \tag{17}$$

together with static boundary conditions

$$\begin{aligned}
&j(\mathbf{n}^\alpha\nu_\alpha) - \bar{\mathbf{t}} = 0\\
&j(\mathbf{m}^\alpha\nu_\alpha) = 0 \quad \text{on } \Gamma_{0\sigma}
\end{aligned} \tag{18}$$

This shell formulation will be used on both the global and the local scale to model the homogenized structure and the RVE, respectively. Global and local quantities will only depend on each other where explicitly specified when the coupling is introduced in the next section. Thus, most quantities must be introduced independently from each other.

In the following, quantities with a bar, such as $\bar{\mathbf{u}},\bar{\boldsymbol{\omega}},\bar{\mathbf{n}}^\alpha,\bar{\mathbf{m}}^\alpha$ will correspond to the global scale system in the shell space Ω_0 with boundary Γ_0. The corresponding variables without a bar, $\mathbf{u},\boldsymbol{\omega},\mathbf{n}^\alpha$, etc. denote quantities on the local scale where a specific RVE i is modeled in the shell space Ω_i with boundary Γ_i, where $i = 1,\ldots,$ *ngp*. Unknown field quantities for both scales are assembled in the vector $\mathbf{v} := [\bar{\mathbf{u}},\bar{\boldsymbol{\omega}},\mathbf{u},\boldsymbol{\omega}]^T$ with variations $\delta\mathbf{v} := [\delta\bar{\mathbf{u}},\delta\bar{\boldsymbol{\omega}},\delta\mathbf{u},\delta\boldsymbol{\omega}]^T$. Assuming a discretization of the global scale system with *numel*