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An analytical solution for pre-crack behaviour of laminated glass under blast loading

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ABSTRACT

Laminated glazing is often employed to minimise damage and injuries during blast events. In this work, the von Karman theory for large deflections of plates was used to simulate the effect of large explosions on laminated glazing. Linear material properties were assumed for both the glass and Polyvinyl Butyral layers. The glass and PVB layers were assumed to act fully compositely during the pre-crack phase of the deformation. A higher order deflection function was employed to represent the complex deformed shape observed in DIC blast test data collected by Hooper et al. (2012). The deflection results showed that the method developed could produce accurate estimates of the glazing deformation history during a blast event. The analytical solution was also used to compute the reaction forces acting on the window supports, which were found to be of a similar magnitude to those calculated from experimental data. In addition, crack densities were predicted, which were found to follow a pattern similar to those seen in blast experiments. The analytical approach developed is valuable for risk assessment engineers and façade designers who much prefer analytically based models over full-scale FE analysis, as FEA is often too time consuming for design assessments.

1. Introduction

The façade system of buildings is of high importance for designers attempting to improve the security of structures against accidental or malicious explosions. The outer layer of the affected buildings needs to prevent blast waves from penetrating the building interior, protecting the occupants and internal fittings. Windows often represent particularly weak areas in this context. Traditional annealed glass is not well suited to protect from blast shock waves, as it tends to fracture early, projecting fragments inside and outside the building and allowing further blast pressure waves to penetrate the building envelope. Laminated glass has been shown to significantly increase the protection of structures [2]. This composite material is formed by two or more layers of glass interposed with layers of Polyvinyl Butyral (PVB), a polymer membrane with good optical properties. When loaded with the pressure wave, the glass layers will tend to fracture. Following this, the PVB membrane is able to stretch, absorbing a significant quantity of incident energy. Additionally, the glass fragments will generally remain attached to the interlayer, preventing them from being flown into the building space.

However, this composite material is difficult to account for in design, as its behaviour is complex. Additionally, the impulsive nature of the loading and the large deflections caused invalidate several simple analytical techniques. To obviate this, single degree of freedom approximations are often used, employing several empirical constants to obtain the required design data [2,3]. To obtain such constants and to study the effect of such loadings on window panes, blast experimental programmes have been carried out in the past [4]. Hooper et al. [1] performed tests where Digital Image Correlation (DIC) techniques were also used to collect deformation data throughout the extent of the glazing pane. These data was used subsequently by Del Linz et al. [5] to calculate the reaction forces acting on the supports and therefore provide further data to determine empirical constants to be used in design. Several other authors [6–10] also developed finite element analysis (FEA) models to perform analyses of the system. Whilst the results obtained are often realistic, the models can become very complex, requiring significant expertise and computing power to be utilised.

Wei and Dharani [11] developed an analytical model to predict the large deformations of the glazing before the failure of the glass

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layers, assuming a first order cosine wave to represent the deflected shape. They used this model to reproduce results from several experiments, which generally utilised relatively small charges. The method was then also extended to calculate likely crack densities in different areas of the pane [12]. Whilst their results agreed well with the experimental data for low explosive quantities, the solution proposed was not able to account for the behaviour shown by the DIC results for heavier blast loadings. In these, the measured deflected shape of the window indicated that a single term solution would not be able to represent the deflections accurately, as higher order cause the deflected shape to differ from a simple sinusoidal deflection in the early phase of the loading [1].

Therefore, in this paper, the analytical solution was expanded to account for these effects. Both the deflection and the Airy's stress function were considered as summations of several terms, and a general solution for any number of terms was derived. This was employed to produce the required differential equations, which were solved numerically. The deflected shapes were then compared with the experimental results. Additionally, the results were used to calculate the stresses acting on the window pane, and from these reaction forces and likely crack densities were found. These again were compared with experimental results available, with the aim of validating the proposed method.

2. Method

Three of the blast tests (Tests 1–3) performed by Hooper on laminated glass were considered for this analysis [1]. One of these tests (Test 1) used 12.8 kg of C4 explosive, (15 kg TNT equivalent), whilst two more tests (Test 2 and Test 3) employed 25.6 kg of C4 explosive, (30 kg TNT equivalent). The charge weights and explosive stand-offs are summarised in Table 1. All the glazing samples were 1.5×1.2 m in planar dimensions and were composed of two 3 mm thick layers of annealed glass and a 1.52 mm PVB layer. Data was collected using pressure gauges, strain gauges and 3D DIC. This last technique allowed the measurements of full field 3D deflections throughout the panes. The experimental set up is shown in Fig. 1, whilst typical DIC results are shown in Fig. 2.

These data collected by Hooper et al. [1] were employed to validate an analytical solution of the plate deformation developed using the Von Karaman theory for large deformations, a commonly used method to represent these phenomena [11,13,14]. The plate was assumed to be thin, therefore through thickness effects were ignored in the analysis. Using this theory, however, required some assumptions both regarding the material properties of the panel and the loading. These will be considered first.

3. Material properties

Laminated glass is composed of two or more layers of glass interposed with layers of Polyvinyl Butyral, a rubbery polymer. Whilst any type of glass could be used, in this analysis annealed glass was assumed, as this was the variety used in the blast experiments.

Different levels of composite deformation theory or action between the composite components were also presumed. However, as in previous studies full composite action was considered a reasonable assumption previous to the glass failure point [1,12]. Therefore, the rule of mixtures could be used to estimate the stiffness and Poisson's ratio of the system. The formulae used were:

Table 1
Blast tests explosive quantities and stand offs.

Test	C4 charge weight (kg)	TNT equivalent charge weight (kg)	Stand-off (m)
1	12.8	15	13
2	25.6	30	16
3	25.6	30	14

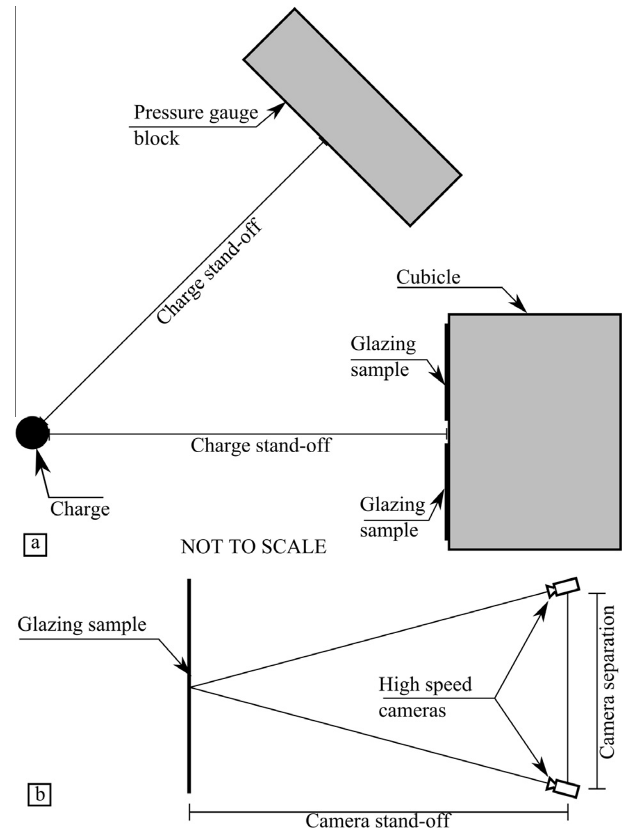

Fig. 1. Typical blast test set up, showing the charge, glazing sample and pressure gauges (a) and the DIC equipment in the cubicle (b). Adapted from [1]).

$$E = \frac{2E_g h_g + E_p h_p}{h_g + h_p} \tag{1}$$

$$\upsilon = \frac{2\nu_g h_g + \nu_p h_p}{h_g + h_p} \tag{2}$$

where E_g and E_p were the Young's modulus of the glass and PVB respectively, ν_g and ν_p were the Poisson's ratios and h_g and h_p were the thicknesses of each layer. The mass per unit area of the section could be found through:

$$M = 2\rho_g h_g + \rho_p h_p \tag{3}$$

where ρ_g and ρ_p were the density of glass and PVB respectively.

Therefore, to apply the formulae above, it was necessary to assume appropriate material properties for each material. Whilst the glass layers would be normally considered as acting in a linear elastic manner previous to failure, the PVB membrane material was significantly more complex to model. Authors performed several experimental studies on this material, which showed significant non linearity and rate dependency in its behaviour [15,16]. Including a material model to accurately represent this complexity in the analytical structural model would have proven impractical. However, before the glass failure, the strains of the PVB membrane were small and the stresses significantly lesser than those in the glass layers, due to the much higher stiffness of the outer material. Because of this, previous authors normally considered the PVB as linear elastic for the purpose of pre-crack analysis [5]. The same