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The analysis of laminated plates using distinct advanced discretization meshless techniques

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ABSTRACT

Composite laminated plates are structural elements with a high strength/weight ratio, being very popular in the aeronautic industry. This work analyses such structural elements using the finite element method, an approximation meshless method (the element free Galerkin method) and three interpolation meshless methods (the radial point interpolation method, the natural neighbor radial point interpolation method and the natural radial element method). Here, the displacement field of the plate is defined by an equivalent single layer theory – the first-order shear deformation theory (FSDT). Thus, a brief theoretical description of the advanced meshless techniques extended to the analysis of composite plates, considering a weak-form approach combined with the FSDT, is presented. In the end, several composite laminates are analyzed and the results obtained with the distinct numerical approaches are compared and discussed. The results show that meshless methods are capable to efficiently analyze composite laminated plates submitted to static loads under linear elastic regimes.

1. Introduction

Laminated composite materials are a class of advanced materials, which have been an important research topic in computational mechanics in the last years. These materials are extensively used by the transportation industry and particularly applied by the aeronautic industry. The main advantage of laminated composite materials are the evidenced exceptional mechanical properties, such as the high strength/weight and stiffness/weight ratios. Being a heterogeneous material, made by a combination of a matrix material and a layered reinforced material, it is important to develop efficient numerical techniques capable to predict accurately the deformation, strain and stresses fields of structural elements using laminated composites materials. These numerical models have a high practical relevance, since they allow to achieve optimal designs for industrial applications.

In this work the laminate composite plate problem is formulated considering the first-order shear deformation theory (FSDT) presented in the early works of Reissner [1] and Mindlin [2], which is an equivalent single layer plate theory assuming first order displacement functions and considering a shear correction factor for attenuating the non-zero transverse shear strain on the top and bottom surfaces.

Nowadays, the finite element method (FEM) is the most popular numerical method in computation mechanics [3] and the most common approximation technique used to analyze composite laminated plates [4]. The FEM divides the continuum solid domain with a finite number of elements, respecting a pre-establish configuration which can be adapted to the discretized solid domain. The simple discretization concept of the FEM is its major advantage. Nevertheless, the FEM is a mesh-based approximation method, which can bring disadvantages in some computational mechanics fields, such as the ones requiring a constant update of the discretization mesh. For instance: in the large deformation problems, the extraordinary mesh distortion decreases the FEM accuracy and the FEM solution stability; in the explicit fluid flow analysis, the constant mesh update increases significantly the computational cost; and in the crack propagation path problem, the moving discontinuity requires a local re-meshing.

Thus, recently, advanced discretization meshless techniques started to arise [5,6]. Capable to answer to some of the FEM drawbacks, the meshless discretization techniques (commonly

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known as meshless methods) present relevant advantages when compared with the FEM [7]. To name a few, the shape functions of meshless methods present virtually a higher order, which permit a higher continuity and reproducibility, and have compact support. Meshless methods are capable to handle problems showing a transient geometry, such as crack propagation and phase transformation problems. Additionally, meshless methods are capable to achieve a more accurate approximation (when compared with low order FEM) and permit to add (or remove) easily nodes to the discretization set, facilitating the refinement procedure.

Nevertheless, FEM and meshless methods are not incompatible, both can be combined to achieve superior computational performances [8].

The generic meshless method formulation permit to discretize the problem domain with an unstructured nodal distribution covering the studied physical domain. In the FEM, the shape functions are constructed at the element level, which is the basic structure defining the nodal connectivity in the FEM. Differently, in the meshless methods, the shape functions are obtained for a flexible set of nodes, known as influence-domain, which can possess a variable size and shape along the discretized domain. It is the overlap of the influence-domains that permit to impose the nodal connectivity and define the field function applicability space [9].

In the literature, it is possible to find several research works extending meshless methods to the analysis of solid mechanics problems using the strong formulation [10,11]. Nevertheless, in this work, only meshless method approaches combined with the Galerkin weak formulation are considered.

Commonly, meshless methods with compact support are divided in two classes: approximation meshless methods and interpolation meshless methods. Approximation meshless methods appeared first, as for example the smoothed particle hydrodynamics (SPH) [12], based in the kernel estimation [13]. The SPH is a meshless method widely used to solve free surface flow problems [14]. Considered by many as the first mature meshless method for solid mechanics, the diffuse element method (DEM) [15] constructs the approximation shape functions using the moving least square (MLS) approximants [16]. Afterwards, Belytschko and coworkers improved the DEM and developed one of the most popular meshless method, the element free Galerkin method (EFGM) [17]. In the same period, the SPH was modified to suit the demands of solid mechanics problems, originating the reproducing kernel particle method [18], and the meshless local Petrov–Galerkin method (MLPG) was developed [19].

Although the proven success in computational mechanics of approximation meshless methods, these numerical techniques are not capable to produce shape functions possessing the delta Kronecker property, which hinders the numerical imposition of essential and natural boundary conditions. At the time, the majority of the meshless method community answered to this drawback researching and developing meshless methods with interpolation functions. Thus, several interpolation meshless methods were created, such as the elegant natural element method [20,21], using the natural neighbor mathematical concept to impose the nodal connectivity and the Sibson interpolation technique to construct the shape functions.

Another very popular interpolation meshless method is the Point Interpolation Method (PIM) [22], capable to be unfolded in several efficient versions [23]. One of this version, and certainly the most applied in computational mechanics, is the Radial Point Interpolation Method (RPIM) [24], which combines the polynomial basis of the PIM with a radial basis function. More recently, the RPIM was combined with the NEM and a new truly meshless RPIM approach was born, the natural neighbor radial point interpolation method (NNRPIM) [25,26,9]. Belinha and co-workers simplified the NNRPIM and delivered an interpolation meshless method combining the low-order connectivity of the FEM with the geometric flexibility of a meshless method [27–29].

In the literature, it is possible to find several studies regarding the analysis of plates and laminated plates using meshless methods. In an early study, Donning and Liu presented an efficient meshless approach to analyze thick plates avoiding the shear-locking effect [30]. Afterwards, the EFGM was combined with the FSDT [31] and several creative numerical techniques to attenuate the shear-locking phenomenon were presented [32–34]. Other meshless approaches were extended to the analysis of thick plates considering the FSDT [35–37,26,28].

Regarding the study of laminated plates using the FSDT, in the literature it is possible to find several meshless studies [38,34,26,29], some of them considering the material non-linear behavior [39].

In this work, the performance of distinct meshless techniques is compared. Thus, a computational framework, capable to analyze composite laminated plates assuming the FSDT, was written in Matlab© for each one of the numerical approaches: FEM, EFGM, RPIM, NNRPIM and NREM. Since all methodologies were programmed following the same layout, the framework permits a more pragmatic comparison regarding the performance and the accuracy of each numeric approach.

2. Meshless methods

The generic meshless method procedure to analyze numerical problems ruled by integro-differential equations is as follows. First, the problem domain $\Omega \in \mathbb{R}^2$, bounded by a physical boundary $\Gamma \subset \Omega$, is fully discretized with a nodal set $\boldsymbol{N} = \{n_0, n_1, \ldots, n_N\}$ scattered in the space domain: $\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N\} \in \Omega$. Then, depending on the meshless method, a nodal dependent, or independent, background integration mesh is constructed. Afterwards, the nodal connectivity is obtained for each interest point (integration point) with the influence-domain concept. The shape functions are constructed using approximation or interpolation functions and then, the global system of equations is established. In this section, a brief description of these steps is presented.

2.1. Nodal connectivity

The nodal connectivity in FEM is established in the preprocessing phase by the numerical construction of a predefined finite element mesh. Thus, the nodes forming each element interact directly with each other. Additionally, the nodes belonging to the geometric boundary of each element interact with the nodes of neighbor finite elements.

In meshless methods there is no predefined nodal connectivity. Instead, the nodal interdependency is enforced with the "influence-domain" geometric construction, which is obtained after the nodal discretization [9]. It is the overlap of the influence-domains that permits to establish the nodal connectivity in several meshless methods.

Generally, influence-domains are obtained by searching radially enough nodes inside a fixed area (2D problems) or a fixed volume (3D problems). Since this technique is very simple to understand (and to implement), it has been used to support the development of several meshless techniques [6,7,9], Fig. 1.

Nevertheless, it has been observed that the performance of the meshless method is influenced by the size or shape variation of these influence-domains along the problem domain [9]. Thus, regardless the used meshless technique, the literature suggests that each 2D influence-domain should possess approximately $n = [9, 16]$ nodes [9]. Both the EFGM and the RPIM use this concept.