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Thermal instability of a nonhomogeneous power-law nanofluid in a porous layer with horizontal throughflow

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ABSTRACT

Despite the experimental observation of shear-thinning rheological behavior depending on the nanoparticle volume fraction, nanofluids were treated as either Newtonian or viscoelastic fluids in previous studies on thermal convection. In this work, taking into account the shear-thinning rheology, Brownian diffusion and thermophoresis of nanofluids, the onset of thermal convection in a nonhomogeneous nanofluid-saturated porous layer with throughflow is investigated. The power-law model is adopted to describe the shear-thinning behavior of nanofluids. The combined effects of Lewis number, Péclet number and power-law index on the thermal instability are analyzed. It is found that the most unstable perturbations are transverse rolls, and both traveling-wave and oscillatory modes may occur. The critical Rayleigh number can be significantly reduced or increased with the increasing power-law index, mainly depending on the value of Péclet number. Furthermore, the decrease of Lewis number can promote or suppress the onset of thermal convection, depending on whether the nanoparticle distribution at the basic state is bottom-heavy or top-heavy.

1. Introduction

The term “nanofluid” was first coined by Choi [1], which refers to the mixture of a base fluid (water, oil, etc.) and a small amount of suspended metallic or metallic oxide nanoparticles (Cu, CuO, Al_2O_3, etc.) with the diameter varying between 1 to 100 nm. Recently, the study of nanofluids has been of growing interest for the observation of its novelty, including the significant enhancement of effective thermal conductivity, abnormal viscosity increase and abnormal single-phase convective heat-transfer coefficient increase relative to the base fluid [2–5]. The heat transfer enhancement of nanofluids may throw light on the urgent cooling problems and thermal energy storage systems in engineering [6].

Several attempts have been made to explain the abnormal characteristic feature of nanofuilds [7–10], but a satisfactory explanation is yet to be found [11]. Buongiorno [12] made an explanation for the abnormal convective heat transfer enhancement observed in nanofluids. He concluded that the Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids, and developed a two-component four-equation nonhomogeneous equilibrium model for mass, momentum, and heat transport in nanofluids. Based on the Buongiorno’s model, thermal convection of nanofluids has received considerable attention. Thermal convection of nanofluids for the Rayleigh–Bénard problem in non-porous media and the Horton–Rogers–Lapwood problem in porous media was discussed by many researchers [13–18]. Those studies treated the nanofluids as either Newtonian fluids or viscoelastic fluids.

However, recent experimental result shows a shear-thinning rheological behavior of nanofluids [19]. For describing the shear-thinning rheology, the power-law model is a good choice

$$\tau = \eta|\dot{\gamma}|^{n-1}\dot{\gamma}, \tag{1}$$

where τ is shear stress, $\dot{\gamma}$ the shear rate, η the consistency factor and n the power-law index. Pak and Cho [20] measured the viscosities of Al_2O_3-water and TiO_2-water nanofluids as functions of the shear rate. Their experimental results reveal that the appearance of the shear-thinning behavior depends on the species of the nanoparticle and the nanoparticle volume fraction. Table 1 shows the values of η and n as a function of volume fraction ϕ for the Al_2O_3-water nanofluid [21].

Although the shear-thinning effect can be obviously observed in experiments and plays an important role in calculation of heat transfer rate [22], there exist few studies on thermal convection of power-law nanofluids. Nield [23] made some discussion on the onset of thermal convection in a porous medium saturated

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by a power-law nanofluid, but no quantitative result was given. On the other hand, in study of thermal convection it is of great significance to determine the criterion for the onset of convection in terms of the critical Rayleigh number. If the Rayleigh number is lower than the critical value, heat transfer is dominated by conduction; but when the Rayleigh number exceeds the critical value, thermal convection sets in and heat transfer rate is enhanced a lot [24].

In this work, taking into account the shear-thinning behavior, Brownian diffusion and thermophoresis effects of nanofluids, we study the thermal convection in a power-law nanoafluid saturated porous layer with throughflow. The critical Rayleigh number and convective modes are determined by performing the linear stability analysis.

2. Formulation of the problem

We consider a horizontal porous layer with thickness H, saturated by a nanofluid. The porous medium is heated from below with a constant temperature difference $\Delta T = T_h - T_c$ across the thickness, and is subject to a stationary, horizontal and uniform throughflow $\mathbf{u}_b$.

2.1. Momentum equation

For a Newtonian fluid, the constitutive relation between extra stress and strain rate is given by

$$\tau = \mu\dot{\gamma}, \tag{2}$$

where μ is the Newtonian viscosity. In modeling flows of a Newtonian fluid in porous media, the simplest but most widely used model in engineering is the well-known Darcy equation

$$\frac{\mu}{K}\mathbf{u} = -\nabla p + \rho\mathbf{g}, \tag{3}$$

where $\mathbf{u}$ is the Darcy velocity, K the permeability, ρ the density of fluid and $\mathbf{g}$ the gravity acceleration.

In the present study, the nanofluid is treated as a power-law non-Newtonian fluid. Consequently, the Darcy Eq. (3) needs to be modified to model power-law fluids in porous media. The power-law constitutive Eq. (1) can be rewritten in the same form of Eq. (2) in terms of an effective viscosity

$$\tau = \mu_e\dot{\gamma}, \tag{4}$$

with the effective viscosity

$$\mu_e = \eta|\dot{\gamma}|^{n-1}. \tag{5}$$

To extend the Darcy equation to modelling flows of a power-law fluid in porous media, it is assumed that the non-Newtonian effect can be subsumed in a suitable definition of μ in Eq. (3) [25]. In view of Eq. (5) and on dimensional grounds, many researchers proposed that the modified μ for power-law fluids in porous media can be given by $\mu = \eta|\mathbf{u}|^{n-1}$ [25–27]. Thus, for a power-law fluid-saturated porous media, the classical Darcy equation is generalized as

$$\frac{\eta}{K^*}|\mathbf{u}|^{n-1}\mathbf{u} = -\nabla p + \rho\mathbf{g}, \tag{6}$$

where η is the consistency factor(SI unit is Pa s^n), K^* is the generalized permeability(SI unit is m^{n+1}).

For a nonafluid, the density ρ in Eq. (6) is defined as

$$\rho = \phi\rho_p + (1-\phi)\rho_f, \tag{7}$$

where ϕ is the volume fraction of nanoparticle, ρ_p the density of nanoparticle, ρ_f the density of base fluid. In light of the Oberbeck–Boussinesq approximation, the buoyancy term in Eq. (6) beomes

$$\rho\mathbf{g} = \left\{\phi\rho_p + (1-\phi)\rho_0[1-\beta(T-T_c)]\right\}\mathbf{g}, \tag{8}$$

where β is the volumetric expansion coefficient and ρ_0 is the reference density of base fluid at temperature T_c.

2.2. Mass conservation equations

The mass conservation equation for nanoparticles in the absence of chemical reactions is given by

$$\frac{\partial\phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{u}\cdot\nabla\phi = -\frac{1}{\rho_p}\nabla\cdot\mathbf{j}_p, \tag{9}$$

where ε is the porosity of porous matrix and $\mathbf{j}_p$ is the diffusion mass flux for the nanoparticles.

The Buongiorno's model treats the nanofluids as a two-component mixture (nanoparticles + base fluid) [12]. Accordingly, relative to the flow velocity $\mathbf{u}$, nanoparticles also display Brownian motion and thermophoresis due to their size on nanoscale. The Brownian motion is proportional to the volumetric fraction of nanoparticles in the direction from high to low concentration, and the thermophoresis is proportional to the temperature gradient from hot to cold. Thus, $\mathbf{j}_p$ consists of two diffusion terms

$$\mathbf{j}_p = \mathbf{j}_{p,B} + \mathbf{j}_{p,T}, \tag{10}$$

where $\mathbf{j}_{p,B}$ and $\mathbf{j}_{p,T}$ denote nanoparticle flux due to Brownian diffusion and thermophoresis, respectively, which can be calculated as

$$\mathbf{j}_{p,B} = -\rho_p D_B\nabla\phi, \quad \mathbf{j}_{p,T} = -\rho_p D_T\frac{\nabla T}{T_c}, \tag{11}$$

where D_B is the Brownian diffusion coefficient and D_T is referred as thermophoretic diffusion coefficient.

With the help of Eqs. (10) and (11), the mass conservation Eq. (9) finally reduces to the following form

$$\frac{\partial\phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{u}\cdot\nabla\phi = D_B\nabla^2\phi + \frac{D_T}{T_c}\nabla^2 T. \tag{12}$$

Furthermore, the continuity equation for the nanofluid is given by

$$\nabla\cdot\mathbf{u} = 0. \tag{13}$$

2.3. Energy equation

The energy equation for nanofluids in porous media can be given by

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\mathbf{u}\cdot\nabla T = -\nabla\cdot\mathbf{q} + \varepsilon h_p\nabla\cdot\mathbf{j}_p, \tag{14}$$

where $h_p = c_p\nabla T$ is the specific enthalpy of nanoparticles. $\mathbf{q}$ is the energy flux relative to the flow velocity $\mathbf{u}$, which can be calculated as [12]

$$\mathbf{q} = -k_m\nabla T + \varepsilon h_p\mathbf{j}_p. \tag{15}$$

Substituting Eq. (15) into Eq. (14) yields

Table 1
The consistency factor η and power-law index n for different volume fraction ϕ [21].

Nanoparticle volume fraction ϕ (%)	η (N sec^n/m^2)	n
0.0	0.00100	1.000
1.0	0.00230	0.830
2.0	0.00347	0.730
3.0	0.00535	0.625
4.0	0.00750	0.540
5.0	0.01020	0.460