Composite Structures 143 (2016) 317–323

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Cylindrical discrete-layer model for analysis of circumferential cracked pipes with externally bonded composite materials

Kwang S. Woo, Jae S. Ahn *, Seung H. Yang

Department of Civil Engineering, Yeungnam University, 280 Daehak-Ro, Gyeongsan, Gyeongbuk 38541, Republic of Korea

ARTICLE INFO

Article history:
Available online 22 February 2016

Keywords:
Composite materials
Cracked pipes
Cylindrical discrete-layer model
Patch repair
VCCT
Circumferential crack

ABSTRACT

This study concerns circumferentially cracked pipes repaired by composite materials subjected to an axial tension. For this purpose, a cylindrical discrete-layer model with a simple and efficient approximation approach is proposed to implement parametric studies on properties of composite materials for patch repair, patch thickness, adhesive shear modulus, adhesive thickness, and thickness of the pipes. Elements of the cylindrical discrete-layer model have a subparametric concept that considers linear mapping of geometry fields on a cylindrical coordinate, and hierarchical approximating functions of displacement fields. The approximating functions are based on one- and two-dimensional integrals of Legendre polynomials, allowing accurate simulation of three-dimensional behavior. Also, a virtual crack closure technique (VCCT) is formulated for the proposed elements to obtain the strain energy release rates (SERR) of pipes with crack-like flaws. The easy-to-use modeling scheme is applied to single-patch repaired pipes with crack. Convergence tests of the elements and sensitivity tests of the modified VCCT are implemented and then some of the present analysis results are verified through comparison with some reference values.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

For patch repair of structures with local damage or defects, composite materials have been preferred due to high strength and stiffness, light weight, and sometimes low cost. In the case of numerical methods for the bonded repaired systems, many researchers [1–4] have relied on the conventional finite element method based on mesh refinement, utilizing hexahedral elements for three-dimensional (3-D) modeling and shell elements for two-dimensional (2-D) modeling. Although there is extensive information on the stress analysis of bonded patch systems in the literature [1–8], most of currently available analysis methods and empirical databases are limited to flat plates. However, curved thin-walled structures are widely used in engineering applications such as aircraft wings, fuselages, and pipe piles of offshore structures. For the analysis of curved repairs, Sun and Tong [9] proposed a curved adhesive element formulation combined with a serendipity plate/shell element, and investigated the effect of curvature on repair effectiveness. Recently, conventional solid elements were applied to the analysis of corrosion-damaged steel pipe piles with curved patches [10] and crack propagation analysis of repaired pipes under cyclic pressure [11].

The conventional finite elements based on mesh refinement would fail to predict smooth and accurate variations of stresses, especially for laminated composite systems such as patch-repaired problems with strong stress singularities. Even if a highly refined mesh is used to model problems having significant stress gradients, the accuracy of stresses obtained by using lower-order finite elements is rather poor [12]. For 3-D models of the bonded patch systems, especially, discretization of the extremely thin adhesive layer with hexahedral elements with acceptable aspect ratios led to models with unacceptably large numbers of elements. This has motivated researchers to develop robust finite elements with the ability to predict not only displacements but also stresses accurately for a wide class of practical problems involving stress singularities. One possibility for robust finite elements is to use higher-order finite elements – this completely eliminates certain types of locking which can lead to poor approximations in engineering application [13].

The aim of the present study is to propose a cylindrical discrete-layer model in order to efficiently simulate the 3-D behavior of adhesively bonded patched pipes. For fracture analysis using the proposed elements, strain energy release rates (SERR) are calculated by the virtual crack closure technique (VCCT) based on linear

* Corresponding author. Tel.: +82 53 810 7850; fax: +82 53 810 4755.
E-mail address: jsahn@ynu.ac.kr (J.S. Ahn).

http://dx.doi.org/10.1016/j.compstruct.2016.02.044
0263-8223/© 2016 Elsevier Ltd. All rights reserved.

elastic fracture mechanics. Lastly, performance of the single-sided patch repair scheme is investigated on patch material, patch thickness, adhesive shear modulus, adhesive thickness, patch thickness and pipe thickness.

2. Cylindrical discrete-layer model

2.1. Geometry fields

A linear mapping on a general Cartesian coordinate would not be accurate enough for laminated cylindrical shells with curved surfaces. To avoid geometrical sources of error, an exact mapping of the curved surface is necessary because one of the sources of efficiency in the present model is the use of as few elements as possible. To describe cylindrical shapes such as pipes with simple linear mapping, geometry fields of elements are defined by nodal modes on cylindrical coordinate. The nodal modes are defined in Section 2.2.

2.2. Displacement fields

For the in-plane variation of approximating functions on curved surfaces, the 2-D hierarchical approximating functions for the nodal, edge, and bubble modes in terms of standard coordinates (ξ,η) are expressed. The nodal modes are the same as linear shape functions defined over a standard square element bounded by $\xi = \pm 1$ and $\eta = \pm 1$. Thus, the shape functions for p-level = 1 are

$$V_i = A^i(\xi)\cdot A^i(\eta), \quad i = 1,2,3,4 \tag{1}$$

$$\text{with} \quad A^i(x) = \frac{1+x_i x}{2} \tag{2}$$

where x_i denotes the local coordinates corresponding to the ith mode. For higher p-levels, additional edge modes must be added for each side and are defined as

$$\left.\begin{aligned} S_{4(l-1)+\widehat{i}+ni} &= A^{\widehat{i}}(\eta)\cdot H_{l+1}(\xi) \quad \text{in } \widehat{i} = 1,3 \\ S_{4(l-1)+\widehat{i}+ni} &= A^{\widehat{i}}(\xi)\cdot \mathrm{H}_{l+1}(\eta) \quad \text{in } \widehat{i} = 2,4 \end{aligned}\right\}, \quad l = 2,\ldots,p \ \text{ and } \ ni = \sum_{\widehat{j}=1}^{i-4>0} \widehat{j} \tag{3}$$

Here, the superscript $\widehat{i} = 1$ in Eq. (3) represents the starting nodal mode number of an edge of the standard element traversing in the counterclockwise direction. For $i > 1$ the integral of the Legendre polynomials is defined as

$$H_{i+1}(\chi) = \frac{1}{i!}\sqrt{\frac{2i-1}{2^{2i+1}}}\int_{-1}^{\chi}\frac{d^i}{dx^i}(x^2-1)^i dx \quad \text{for } i = 2,\ldots,p \tag{4}$$

In addition to above, for $p \geqslant 4$, $\frac{1}{2}(p-2)(p-3)$ shape functions corresponding to internal modes need to be added. These can be defined as

$$I_k(\xi,\eta) = H_{i+2}(\xi)H_{l+2-i}(\eta), \quad i = 1,2,\ldots,l-3;$$
$$k = 4l + \sum_{ni=0}^{p-4} ni + i; \quad l = 4,5,\ldots,p \tag{5}$$

In the present model, the 3-D behavior is explained by defining 2-D approximating functions (p-level) in both the bottom and top surfaces of a typical layer l. The displacement variations between these two curved surfaces are then accounted for by hierarchically appending 1-D approximating functions (q-level) that enable a higher-order variation of displacement across the thickness of a typical layer. The approximating functions are called as thickness modes. The displacement field with any p- and q-levels in any layer is then given by

$$\begin{aligned} u_{x,\theta,r} = {} & V_i\left[\frac{1-\zeta}{2}(\alpha_i^{bot})^c + \frac{1+\zeta}{2}(\alpha_i^{top})^c + H_{\bar m}(\alpha_i^{\bar m})^c\right] \\ & + S_j\left[\frac{1-\zeta}{2}(\phi_j^{bot})^c + \frac{1+\zeta}{2}(\phi_j^{top})^c + H_{\bar m}(\phi_j^{\bar m})^c\right] \\ & + I_k\left[\frac{1-\zeta}{2}(\gamma_k^{bot})^c + \frac{1+\zeta}{2}(\gamma_k^{top})^c + H_{\bar m}(\gamma_k^{\bar m})^c\right] \quad \text{in } \bar m = 3,4,\ldots,(q+1) \end{aligned} \tag{6}$$

where $-1 \leqslant \xi \leqslant +1$ is the standard coordinate in the thickness direction of a layer.

2.3. Constitutive equations

If the cylindrical shells are made of linearly cylindrical orthotropic materials that have three planes of symmetry that coincide with the cylindrical coordinate representation, for a typical layer, the orthotropic stress–strain relations are given by

$$\langle\sigma_{x,\theta,r}\rangle^{\mathrm{T}}_{1\times 6} = [\tilde D]^l_{6\times 6}\langle\varepsilon_{x,\theta,r}\rangle^{\mathrm{T}}_{1\times 6} \tag{7}$$

where

$$\sigma^l_{x,\theta,r} = \langle\sigma_{xx}\ \sigma_{\theta\theta}\ \sigma_{rr}\ \sigma_{\theta r}\ \sigma_{rx}\ \sigma_{x\theta}\rangle^{\mathrm{T}}; \quad \varepsilon^l_{x,\theta,r} = \langle\varepsilon_{xx}\ \varepsilon_{\theta\theta}\ \varepsilon_{rr}\ \varepsilon_{\theta r}\ \varepsilon_{rx}\ \varepsilon_{x\theta}\rangle^{\mathrm{T}} \tag{8}$$

Also, the elasticity matrix $[\tilde D]^l_{6\times 6}$, in terms of the element coordinate system, is derived from the elasticity matrix $[D]^l_{6\times 6}$ with respect to the orthotropic material axes by using the coordinate transformation matrix $[H]_{6\times 6}$, such that

$$[\tilde D]^l_{6\times 6} = [H]^{\mathrm{T}}_{6\times 6}[D]^l_{6\times 6}[H]_{6\times 6} \tag{9}$$

In Eq. (8), the strains are described by strain–displacement relations as follows:

$$\begin{aligned} &\varepsilon_{xx} = \frac{\partial u_x}{\partial x}; \quad \varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}; \quad \varepsilon_{rr} = \frac{\partial u_r}{\partial r} \\ &\varepsilon_{\theta r} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}; \quad \varepsilon_{rx} = \frac{\partial u_x}{\partial r} + \frac{\partial u_r}{\partial x}; \quad \varepsilon_{x\theta} = \frac{1}{r}\frac{\partial u_x}{\partial \theta} + \frac{\partial u_\theta}{\partial x} \end{aligned} \tag{10}$$

2.4. Virtual work statement

The displacement fields $\{\Phi\}$ of a layer with components defined in Eq. (6) can be expressed in the following compact form:

$$\{\Phi\} = [\tilde H]\{a\} + [\tilde G]\{b\} \tag{11}$$

where the matrices $\{\tilde H\}$ and $\{\tilde G\}$ are in terms of approximating functions corresponding to nodal modes $\{a\}$ related to nodal modes and non-nodal modes $\{b\}$ including edge, bubble, and thickness modes, respectively. The nodal modes have physical meaning, while non-nodal modes depending on order of the hierarchical approximating functions do not have physical meaning but improve accuracy of analysis. The element equations for a layer can be expressed by using the principle of virtual work as follows:

$$\int_V \delta\{\tilde\varepsilon\}^{\mathrm{T}}\{\tilde\sigma\}dV - \delta W = 0 \tag{12}$$

where δW is external virtual work. Virtual displacements can be expressed as

$$\delta\{\Phi\} = [\tilde H]\delta\{a\} + [\tilde G]\delta\{b\} \tag{13}$$

and the corresponding virtual strain can then be expressed as

$$\delta\{\tilde\varepsilon\} = [\tilde B]\delta\{a\} + [\tilde N]\delta\{b\} \tag{14}$$

Using these definitions based on the virtual work, the element stiffness matrix of a layer can be expressed as

Download English Version:

https://daneshyari.com/en/article/6705807

Download Persian Version:

https://daneshyari.com/article/6705807

Daneshyari.com