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Short Communication

Sedimentation of a rotating sphere in a power-law fluid

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ABSTRACT

We measure the sedimenting velocity of rigid spheres in power-law fluids. By imposing a controlled rotation, we can increase the typical shear rate in the surrounding fluid leading to a decrease of the effective fluid viscosity and, consequently, an increase of the sedimentation speed of the spheres. By fitting our experimental measurements to a power-law dependence of the sedimentation speed on the rotation frequency we are able to predict the values of the consistency and power indices for the test fluids. This setup could thus be used as a rheometer for power-law fluids.

1. Introduction

The sedimentation of solid spheres is a classical benchmark problem in fluid mechanics and has therefore been studied extensively for both Newtonian [1] and non Newtonian [2] liquids. One important practical aspect of the study of sedimentation is that it can be used to determine fluid properties. For an infinite Newtonian fluid in the creeping flow regime, the sedimentation velocity of a sphere, U_s, is obtained analytically [3] as

$$U_s = \frac{W^*}{3\pi D\mu}, \tag{1}$$

where D is the sphere diameter, μ is the dynamic viscosity and $W^* = \frac{\pi}{6}(\rho_p - \rho)gD^3$ is the effective weight of the sphere (ρ denotes the fluid density while $\rho_p > \rho$ is the density of the sphere). If the size and weight of the sphere are known and the sedimentation velocity is measured, the value of the viscosity can be inferred from Eq. (1). This method has been used extensively to determine the viscosity of Newtonian viscous fluids [1].

For the particular case of inelastic fluids with shear-dependent viscosity, it is common to express their effective viscosity, μ_{eff}, as a power-law

$$\mu_{\text{eff}} = m\dot{\gamma}^{n-1}, \tag{2}$$

where $\dot{\gamma}$ denotes the flow shear rate and m and n are the consistency and power indices, respectively [4]. Whereas m characterizes the magnitude of the viscosity, n describes how rapidly the viscosity changes with shear rate.

There have been numerous studies aiming to calculate the drag force on solid spheres moving at constant speed in inelastic shear-thinning fluids, as summarized in Chapter 3 of Ref. [2]. The effect of thinning-viscosity for creeping flow regimes is typically expressed empirically as

$$F_D = 3\pi D U_s \mu_{\text{eff}} Y(n), \tag{3}$$

where $Y(n)$ is a drag correction factor defined as $Y(n) = C_D Re/24$, where C_D is the drag coefficient and $Re = \rho D^n U_s^{2-n}/m$. In Eq. (3), the value of μ_{eff} is to be evaluated at the mean shear rate, $2U_s/D$. Despite many investigations, the reported trends for $Y(n)$ vary widely [2]. With advances in numerical simulations, the situation has improved [5–7] but a general consensus on this issue has not been reached. As a result, very few studies have attempted to use the sedimenting sphere setup to measure non-Newtonian fluid properties [8–11]. One difficulty is that the flow around a sphere is not viscometric, and thus average values of the shear rate and stress need to be considered, and rigorous predictive modeling is not available.

In this note, we study the classical problem of a sedimenting sphere but with a twist – literally. Using a recently developed method we are able to impose a rotation, at a controlled rate, around the axis of sedimentation of a rigid sphere. The flow around the sphere is then the combination of that produced by translation

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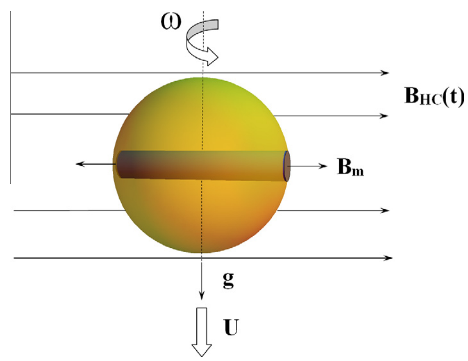

Fig. 1. Sketch of the sphere in the sedimentation experiment and its interaction with the magnetic field.

and that produced by rotation of the sphere. As a result of the particle rotation, the effective viscosity around the sphere decreases, resulting in a decrease of the fluid drag and an increase of the terminal velocity of the sphere.

The study presented here can, in fact, be placed in the subject of flow superposition, a topic which has been addressed by many researchers in the past. For the particular case of non Newtonian and complex liquids, the superposing rotation and translation has been addressed by [12–14]. Other authors have studied the superposition of squeezing flows with rotation [15,16]. For most of these investigations, the motivation was to observe the response of a primary flow to a secondary orthogonal flow. In this sense, our study is similar: we observe the changes in the flow around a sphere which result from imposing shear by rotation. However, for the case addressed in the present study the flow is more complex since sedimentation and rotation are not orthogonal and, therefore, they cannot be fully decoupled. Gheissary and van den Brule [17] and Ovarlez et al. [18] have both previously addressed the superposition of these two flows. To our knowledge, however, the idea to use the superposition of rotation and sedimentation to determine the fluid properties has not been proposed before.

We conducted measurements using four different fluids with different values of the consistency and power indices, m and n, respectively. We found that the velocity of the sedimenting sphere is proportional to ω^{1-n}, where ω is its rotational speed. With this new experiment, we are able infer the fluid properties of shear-thinning inelastic fluids, avoiding the unresolved questions arising in the classical sedimentation situation.

2. Description of the experiment and test fluids

Plastic spheres with diameters $D = 8.28$ mm were used. One or more small rare Earth rod magnets (Magcraft, models NSN0658) with a diameter of 3.18 mm were inserted into a plastic sphere as shown schematically in Fig. 1. The magnets were aligned horizontally, so as to induce a rotation around the sedimentation axis. For the tests shown in this paper, we used two spheres with weights of 0.33 and 0.42 g.

A sphere was released to sediment freely in the middle of a cylindrical container of height 300 mm and diameter 53.4 mm filled with the test fluid. The container was placed in the middle of a device able to generate a rotating magnetic field of constant intensity. Details of the magnetic setup can be found in Ref. [19]. Briefly, the device produces a uniform magnetic field in the entire container. If the direction of the magnetic moment of the magnet is different from that of the external magnetic field a torque is produced, resulting in rotation of the sphere. As the external magnetic field is rotating around the sedimentation axis, the sphere rotates at the same rate and in the same direction.

Table 1
Physical properties of the all fluids tested in this investigation. The ingredients for al fluids (water (W), glucose (G), ethylene glycol (EG), Carbopol (C) and triethyiamine (TEA)) are indicated in percentage by weight. The parameters n and m are the power and consistency indices, respectively, of the power law model. $\mu = m\dot{\gamma}^{n-1}$. The rheological measurements were conducted for $0.1 < \dot{\gamma} < 100\ \mathrm{s}^{-1}$.

Fluid	Composition	n (–)	m (Pa s^n)	ρ (kg/m^3)
N, (□)	G/W, 91/9	0.98	0.602	1390
ST1, (▼)	EG/C/TEA, 97.91/0.06/2.03	0.87	0.15	1116
ST2, (◆)	EG/C/TEA, 98.88/0.10/0.02	0.70	1.031	1113
ST3, (●)	EG/C/TEA, 99.27/0.70/0.03	0.47	3.979	1110

For the test fluids, we fabricated three shear-thinning fluids and a reference Newtonian solution. These fluids are solutions of Carbopol (C) in ethylene glycol (EG). To modify the magnitude of the viscosity and the value of the power index, n, the composition of the solutions was varied. Small amounts of triethyiamine (TEA) were also used to modify the pH of the solutions and vary further the fluid properties. The Newtonian reference fluid was obtained from a solution of glucose and water.

The rheological characterization were conducted using a *TA Instruments AR1000N* rheometer with a cone-plate geometry (60 mm, 2°, 65 μm gap). The physical properties of the solutions are summarized in Table 1. All three non-Newtonian fluids follow closely a power-law behavior in steady shear, with negligible elasticity in the range of shear rates attained in the experiments ($\dot{\gamma} < 100\ \mathrm{s}^{-1}$.) The flow curves look very similar to those of [20], where the same type of fluids were used. We further conducted some oscillatory tests to confirm that elastic effects were negligible, following the scheme proposed by [21]. The composition of the Newtonian reference solution was chosen to have a shear viscosity of the same order as that of the shear thinning fluids.

3. Analysis

We consider a rigid sphere of radius $a = D/2$ sedimenting under its own weight, denoted W^*, along the z direction. In addition to solid-body translation at speed $\mathbf{U} = U_s\mathbf{e}_z$, the sphere is assumed to rotate with angular speed $\mathbf{\Omega} = \omega\mathbf{e}_z,\ \omega > 0$ around the sedimentation axis.

A simple estimate for the sedimentation speed, U_s, can be found by balancing the weight of the sphere with a Stokes drag with an effective viscosity μ_{eff}. Therefore, we can write

$$U_s = \frac{W^*}{3\pi D\mu_{\mathrm{eff}}}. \quad (4)$$

Clearly, the sedimentation velocity will depend on the value of the effective viscosity, μ_{eff}, which in turn will depend on the rheology of the surrounding fluid:

$$\mu_{\mathrm{eff}} = f(\dot{\gamma}, n, \mu_0, \mu_\infty, N_1, N_2, \ldots), \quad (5)$$

where $\dot{\gamma}$ is the shear rate in the fluid, n is the power index, μ_0 and μ_∞ are the zero-shear-rate and high-shear-rate viscosities, respectively, N_1 and N_2 are the first and second normal stress differences, respectively. Of course, these properties will depend on the nature of the fluid rheology and the model used to characterize it; μ_{eff} would embed all the fluid effects of the sedimenting motion. A general closed form of the effective viscosity is not readily available.

For the case of shear-dependent viscosity fluids, μ_{eff} is given by Eq. (2). Therefore,

$$U_s = \frac{W^*}{3\pi D m\dot{\gamma}^{n-1}}. \quad (6)$$