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A variable-kinematic model for variable stiffness plates: Vibration and buckling analysis

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ABSTRACT

This paper presents an advanced approximate technique for the vibration and buckling analysis of variable stiffness plates. The formulation is based on a variable-kinematic approach and is developed in the context of a variational framework together with the method of Ritz. Any set of boundary conditions can be accounted for, while loading conditions of pure axial compression are assumed. Results are validated against finite element predictions and solutions available in the literature, demonstrating the accuracy of the proposed method in terms of eigenvalues and modal shape descriptions. A novel set of vibration and buckling results is provided for moderately thick variable stiffness plates, including monolithic and sandwich configurations.

1. Introduction

In the past years, increasing interest has been devoted to the study of variable stiffness panels. As compared to classical straight fiber configurations, the adoption of a continuous variation of the stiffness properties can provide significant advantages. Indeed, the increased number of design variables extends the tailoring capabilities offered by composites, and requirements on stiffness, buckling and vibration behavior can be strongly improved. One of the first investigations focusing on the potential benefits due to the stiffness variation is found in the work of Leissa and Martin [1]. The paper illustrates the possibility of improving the buckling load and the fundamental frequency by as much as 38% and 21%, respectively. Solutions are derived using thin plate theory and the method of Ritz, and stiffness variation is achieved by means of non-uniformly spaced fibers. Another approach to obtain stiffness tailoring consists in varying the plate thickness. DiNardo and Lagace [2] investigated this strategy, and presented a Ritz-based methodology, in the context of thin-plate theory, to assess the bucking and post-buckling behavior.

More often, stiffness tailoring has been achieved by considering curvilinear fiber configurations, which are also the subject of the present investigation. Back to the pioneering works of Hyer and Charette [3,4] and Gürdal [5–7] and co-workers, many investigations have regarded the development of novel experimental, numerical and analytical methods to handle panels with curvilinear fibers. A comprehensive review can be found in Ref. [8].

Concerning the free vibrations, the number of investigations is relatively restricted and, in most cases, is based on a finite element approach. Abdalla et al. [9] proposed an optimization procedure to achieve the maximum fundamental frequency. The analysis is performed with finite elements based on Classical Lamination Theory (CLT), while lamination parameters are adopted to parametrize the design. The maximum frequency design of conical shells is the subject of Ref. [10], where finite element computations are performed with the commercial code Abaqus, and four node shell elements are used.

Honda and Narita [11] adopt eight-node, first-order finite elements in conjunction with genetic algorithms to maximize the fundamental frequency of locally anisotropic plates, including short and continuous curvilinear fibers. Natural frequencies and modes shapes are presented in the work of Akhavan and Ribeiro [12] using third-order shear deformation theory and taking manufacturing considerations into account. Results are presented for different combinations of boundary conditions, and are a useful benchmark for comparison purposes. In a recent paper, Tornabene et al. [13] presented high-order solutions for singly and doubly-curved panels with curvilinear fibers by means of the Local Generalized Differential Quadrature method.

The relatively large literature dealing with the buckling behavior of variable stiffness panels covers a number of numerical and semi-analytical investigations and, in most cases, CLT is adopted. An early work of Ref. [7] demonstrates the improvements in the

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buckling load thanks to the use of curvilinear fibers, and the implementation of the Ritz method is discussed in the context of CLT. Traditional finite element solutions, as well as highly efficient numerical techniques based on the Ritz technique, are established in Ref. [14].

The Galerkin method is implemented in Ref. [15] to solve the partial differential equations governing the buckling of thin variable stiffness plates. Setoodeh et al. [16] proposes the use of the reciprocal approximation for the buckling maximization of plates subjected to combined loading conditions. The method makes use of finite element computations based on CLT.

Semi-analytical solution strategies for the buckling analysis of variable angle tow panels, modeled using classical lamination plate theory are presented in Refs. [17,18] and are based on the differential quadrature method (DQM) and the method of Ritz, respectively. The Ritz formulation is extended to the analysis of the post-buckling response in Ref. [19], where a mixed variational principle, expressed in terms of out of plane displacement and Airy stress function, is adopted [20,21].

Among the few semi-analytical procedures for variable stiffness panels based on first-order theory, the works of Coburn at al. [22,23] are here mentioned. They rely on the method of Ritz, where the stress function and the out of plane displacement are expanded using Legendre polynomials. The buckling of simply-supported blade stiffened panels is the subject of the research paper of Ref. [22], whereas sandwich plates with variable stiffness face-sheets are considered in Ref. [23]. This second study highlights the importance of the core shear modulus, illustrating that, below a threshold value, no tangible improvements can be achieved with respect to straight fiber configurations.

Despite the relatively large number of research studies dealing with the buckling and vibrations of variable stiffness plates, analytical and semi-analytical techniques have been mainly restricted to CLT and first-order theories. As a matter of fact, high-order solutions have been widely derived for straight fiber laminates using various approaches (see, for instance, [24–26]). In this context, a powerful approach to automatically handle a large variety of plate theories, including equivalent layer and layerwise theories of different order, is the so-called Carrera's Unified Formulation (CUF) [27,28]. Within this framework, the exact solutions of the equations governing the buckling of simply-supported, cross-ply plates is discussed in Refs. [29,30], where the Navier and Lévy methods are applied.

Another interesting application of CUF regards its combined application with approximate techniques, an example of which is given by radial basis functions [31–33]. In these cases, strong form equations and boundary conditions are derived starting from the Principle of Virtual Displacements (PVD); the discretization is then performed on the basis of the interpolation technique of Ref. [34]. An application to the static and free vibration analysis of isotropic and cross-ply plates is found in Ref. [31], while buckling, bending and vibration response of functionally graded sandwich plates is discussed in Refs. [32,33]. Recently, the combined use of radial basis functions and CUF has been extended to the buckling analysis of thin-walled beams [35].

Two other approximate techniques that have been successfully applied in the context of CUF are the Galerkin and the Modified Galerkin methods. They are applied in Refs. [36,37] to obtain an extensive set of buckling and thermo-mechanical buckling solutions for multilayered, simply-supported plates.

Still in the context of straight fiber plates, variable-kinematic formulations have been developed by solving the weak form equations with the method of Ritz. Examples can be found for the buckling [36,38] and vibration [39,40] of composite plates.

To the best of the authors' knowledge, high-order solutions are still quite rare in the case of curvilinear fiber panels. The need for high-order solutions is additionally motivated by the fact that variable stiffness plates were found to be more affected by transverse shear effects than corresponding quasi-isotropic configurations [41]. Furthermore, large part of the works limits the analysis to simply-supported boundary conditions, while free and clamped conditions – and combinations of them – have been rarely assessed.

The present work aims to fill these gaps by presenting a novel semi-analytical approach for the analysis of variable stiffness panels. The theory relies on the use of CUF applied in conjunction with the method of Ritz. The resulting variable-kinematic approach is here denoted as vk-Ritz method.

Based on previous works by the authors for straight fiber panels [30,38,40,42], the semi-analytical model has been extended to account for a continuous variation of the orientation angles in the space. The problem is developed by adopting a displacement-based approach, where the three components of the displacement field are expanded using Chebyshev polynomials and boundary characteristic functions. In the context of a variational framework, the set of equations governing the discrete problem is obtained for the buckling and free vibration analysis.

The vk-Ritz method is validated against results available in the literature, and is here adopted to present reference solutions, using theories of various order, for the buckling and vibrating modes of sandwich and monolithic variable stiffness panels.

2. Variable stiffness panels

The methodology is developed for the analysis of composite plates obtained by the stacking of plies with non-straight fibers; both thin and moderately thick plates are considered. A sketch of the panel is reported in Fig. 1, where the reference system and the dimensions of the plate are illustrated. In particular, the panel is characterized by length a and width b. It is obtained by the stacking of a number N_l of plies, each one characterized by thickness h_k, for a total thickness equal to h.

A Cartesian coordinate system is taken such that the x-axis is directed parallel to the longitudinal edge of length a, and the y-axis is parallel to the transverse edge of length b. The four sides of the panel are numbered in a counterclockwise direction, as reported in Fig. 1.

The four panel edges can be subjected to any combination of clamped, simply-supported and free boundary conditions, while the loading case, for the buckling analysis, is restricted to the pure axial compression.

Different approaches can be assumed to express the fiber angle variation: Lobatto and Lagrange polynomials [18,43], NURBS [44], or linear interpolation within control points [10] are few but examples. In this study, the fiber angle is allowed to vary along the x or the y direction with a linear law, but it is never function of both the coordinates. Therefore, stiffness variation can be achieved along

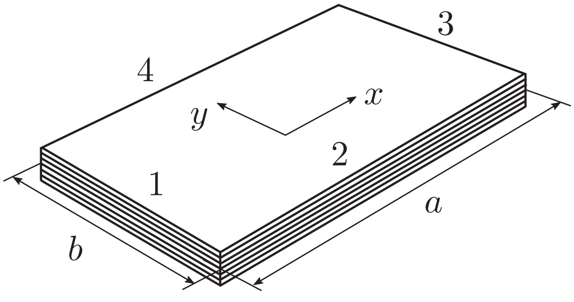

Fig. 1. Multilayered plate dimensions and reference system.