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Transient analysis of stationary interface cracks in orthotropic bi-materials using oscillatory crack tip enrichments

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ABSTRACT

The problem of an interface crack between two orthotropic layers under dynamic loading is analyzed. Special crack tip enrichment functions are incorporated into the standard finite element shape functions to exactly reproduce oscillatory stress and displacement fields near the tip of the interface crack. Moreover, kinematics of displacement and its gradients across the crack face and material interface are also modeled by partition of unity enrichments. Special attention is given to extraction of stress intensity factors by utilizing a proper form of the interaction integral for orthotropic bi-materials. Advantages of this method of extracting stress intensity factors over the conventional displacement extrapolation technique are discussed. Several bi-material configurations with both vanishing and non-vanishing oscillatory indexes are solved using the interaction integral and the results are compared with the available data in the literature. Effects of employing oscillatory crack tip enrichments and validation of the path-independent J-integral are also discussed.

1. Introduction

Combination of individual constituents to utilize the advantages of each material has made composite materials ubiquitous in different branches of engineering, from aerospace and automotive structures to electronic packages. Nonetheless, the inherent weakness associated with the interface joining each material might limit the use of composites, particularly in extreme loading conditions. In the engineering community, interface cracks have been considered as one of the main sources of failure in composite materials, making an accurate analysis of these particular defects necessary. As many applications of composite materials include time-dependent and impact loadings, the current study is focused on analysis of interface cracks under dynamic loadings.

In case of fiber reinforced composite materials, the interface crack is located between layers that are not isotropic, causing a complex stress and displacement field near the tip of an interface crack. Since arbitrary loadings and geometries encountered in a practical engineering problem inhibit the use of analytical methods, numerical techniques must be employed to tackle these problems. A numerical technique widely used in the literature to analyze problems with singularity and discontinuities is the boundary element method (BEM), which makes use of fundamental solutions to accurately simulate complex problems. However, since the fundamental solution for general layered anisotropic media does not exists, Refs. [1–5] utilized the solution of homogeneous anisotropic media in a multi-domain BEM framework to analyze interface cracks under static loading condition. In addition to BEM mentioned above, another numerical technique for numerical analysis of fracture problems is the meshfree method (Refs. [6,7]).

For the cases of dynamic loading, the situation becomes more complicated and only a limited number of investigations can be found [8,9], which suffer from poor stability in the time domain, as mentioned in Ref. [10]. A more stable algorithm was presented by Lei et al. [10], but this work neglected the oscillatory crack tip fields near interface cracks. Song et al. [11] made an improvement and utilized the scaled boundary finite element method to reproduce the mentioned oscillatory fields. It should be also noted that nearly all mentioned BEM-based works utilize the conventional displacement extrapolation technique to extract stress intensity factors, which further requires accurate simulation of crack tip fields. In the case of interface cracks, however, obtaining accurate crack tip displacements is cumbersome, if not impossible. Also, Domain discretization in scaled boundary elements also requires especial care (Refs. [12,13]).

Finite element method (FEM), on the other hand, can readily deal with arbitrary geometries, and many unconditionally stable time integration algorithms do exist for analysis of dynamic problems.

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It is also well suited for post-processing evaluation of J-integral, by which the stress intensity factors can be easily obtained. Nevertheless, while the standard finite element method proved to be successful in common fracture mechanics simulations, the fundamental complications associated with oscillatory stress and displacement fields near the tip of an interface crack, especially in the case of layered orthotropic media, result in its low performance and accuracy. Recently, an algorithm called "the edge rotation algorithm" was proposed for fracture analysis based on FEM. In this simple algorithm, an alternative method to the methods based on enrichment techniques was proposed (Refs. [14,15]).

A remedy is the extended finite element method (XFEM) [16,17], by which the exact crack tip fields can be precisely simulated. Preserving all advantages of FEM over BEM, the XFEM formulation presented here reproduces the exact oscillatory crack tip fields. It also represents the kinematics of strong and weak discontinuity easily. XFEM has been successfully used for static and dynamic fracture analysis of homogeneous orthotropic materials [18–25], and static fracture analysis of isotropic and orthotropic bi-materials [26–28]. As many applications of composite materials include time-dependent and impact loadings, the current study is focused on analysis of interface cracks under dynamic loadings. Therefore, an existing XFEM methodology [27] is further extended to dynamic analysis of interface cracks in orthotropic bi-materials, with particular attention to the effects of oscillatory fields and the interaction integral. Also, it should be noted that prior to the present methods, FEM and BEM solutions developed for analysis of interface cracks under dynamic loadings without the need for any exact solution for distribution of stress and displacement fields around a crack tip. In the current study, however, the exact analytical distribution of displacement and stress fields are introduced for the first time in the dynamic analysis of interface crack and the differences between the proposed method and other numerical methods are demonstrated.

The organization of this work is as follows: First, a description of basics of the considered problem and essential backgrounds of interface fracture mechanics under the assumption of linear elastic fracture mechanic (LEFM) are provided. The next section explains how strong and weak discontinuities and oscillatory crack tip fields are modeled in the context of XFEM. Then, a brief review of the discretized equations and numerical evaluation of crack tip parameters are presented, followed by several numerical simulations to study the effects of employed crack tip enrichments and the interaction integral method. The paper is closed with the concluding remarks.

2. Problem statement

As illustrated in Fig. 1, the elastodynamic problem of a layered orthotropic body containing an interface crack is considered in this study:

$$\mathrm{Div}(\boldsymbol{\sigma}) + \mathbf{f} = \rho \ddot{\mathbf{u}} \tag{1}$$

with the following boundary conditions

$$\mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x}, t) \quad \text{on } \Gamma_u \tag{2}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \Gamma_t \tag{3}$$

and the initial conditions

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0 \tag{4}$$

$$\dot{\mathbf{u}}(\mathbf{x}, 0) = \dot{\mathbf{u}}_0 \tag{5}$$

The Hook's law in plane stress conditions for both linear elastic orthotropic materials are [29]

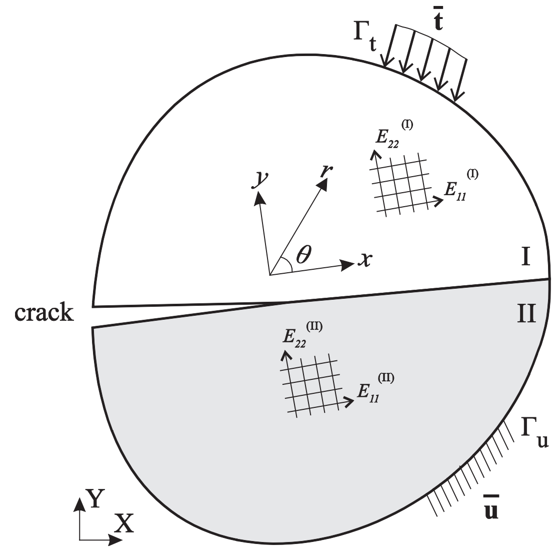

Fig. 1. An interface crack in a layered orthotropic material subjected to dynamic loading.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{26} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & 2D_{1112} \\ D_{2211} & D_{2222} & 2D_{2212} \\ 2D_{1211} & 2D_{1222} & 4D_{1212} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{26} \\ \sigma_{12} \end{Bmatrix} \tag{6}$$

In plane strain conditions, the following replacement should be done:

$$a_{ij} \rightarrow a_{ij} - \frac{a_{i3} - a_{3i}}{a_{33}} \quad i, j = 1, 2, 6 \tag{7}$$

The differential equation including equilibrium, orthotropic constitutive law, and compatibility equation has a characteristic equation of the form [30]

$$t^4 + 2B_{12}t^2 + K_{66} = 0 \tag{8}$$

with

$$B_{12} = \frac{2a_{12} + a_{66}}{2a_{11}} \text{ and } K_{66} = \frac{a_{22}}{a_{11}} \tag{9}$$

Most of the orthotropic materials, including all simulated problems, have properties that ensure $\sqrt{K_{66}} < B_{12}$, resulting in purely imaginary roots of Eq. (8), in the form of

$$t_1 = ip; \; t_2 = iq \quad \text{if } \sqrt{K_{66}} < B_{12} \tag{10}$$

with

$$p = \sqrt{B_{12} - \sqrt{B_{12}^2 - K_{66}}}; \quad q = \sqrt{B_{12} + \sqrt{B_{12}^2 - K_{66}}} \tag{11}$$

For further information and discussion on various types of composites, see Ref. [27].

3. Interface crack mechanics

Consider an interface crack located between two orthotropic layers. The definition of stress intensity factors for interface cracks, proposed by Cho et al. [2], is adopted. Based on this definition, the singular stress at a distance r ahead of the crack tip can be written as

$$[\,\tau_{xy} \quad \sigma_{yy}\,]_{r,\theta=0} = \frac{1}{\sqrt{2\pi r}} \mathrm{Re}\left[\mathbf{KW}\left(\frac{r}{L}\right)^{i\varepsilon}\right] \tag{12}$$