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Time-dependent stress variations in symmetrically viscoelastic composite laminates under uniaxial tensile load

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ABSTRACT

In this paper, we report an analytical approach for time-dependent stress variations in symmetrically viscoelastic composite laminates under plane deformation state based on a stress function based equivalent single layer theory. The theory initially adopts the stress function separated by the in-plane stress function and out-of-plane stress function as field assumption, instead of using the displacement field. The viscoelastic properties are simply expressed by the Maxwell model for the analysis of relaxation effect on the free edge stresses in viscoelastic laminates. The constitutive equation in the integral form for linear viscoelastic materials under constant uniaxial strain load can be simplified. By taking the principle of complementary virtual work, the governing equation can be obtained and further solved by solving a general eigenproblem. Convergent stress distributions are obtained and validated by the 3-D finite element method using commercial package. The free edge stresses are function of time and loading conditions in viscoelastic composite laminates and the relaxation effect on the free edge stresses is clearly shown in the numerical results of viscoelastic composite laminates with various layup stacking sequences.

1. Introduction

Fiber-reinforced composites [1] have become important engineering materials and been widely employed in engineering fields, such as vehicles, aircrafts, sports equipments and so on. As it is well known that one of the biggest disadvantages of fiber-reinforced composite laminates is the mismatch of material properties between adjacent layers, this defect is critical to the service life of composite laminates since the mismatch of material properties could result in severe stress concentrations and complex stress state at layer interfaces even under a simple plane loading condition [2,3]. This defect has been comprehensively studied [4,5] and well understood with a large quantity of numerical and analytical approaches developed for mathematical modeling. But, the previously developed approaches only consider the ones that the laminates have constant elastic properties which are normally for perfect solids. However, when the laminates have viscoelastic motion in a certain environment of high temperature and high moisture, the viscoelastic behavior [6,7] cannot be neglected in structural analysis and the time-dependent properties have to be incorporated into the constitutive equations for viscoelastic laminates.

There are many studies dedicated to the analysis of viscoelastic behaviors, especially for vibration, bending and static stress analyses. Kim [8] studied the nonlinear vibration of viscoelastic laminated composite plates based on von Karman's nonlinear deformation theory and Boltzmann's superposition principle. He investigated the effect of large amplitude on the dissipative nature and the natural frequency of viscoelastic laminated plates. Eshmatov [9] also studied the nonlinear vibrations of viscoelastic orthotropic plates as well as the dynamic stability, but based on the Kirchholff-Love hypothesis and Reissner–Mindlin generalized theory. Reddy [10] recently reported a work of nonlinear viscoelastic analysis of orthotropic beams using a general third-order theory with the finite element implementation to study the quasi-static behavior. Nguyen et al. [11] reported an efficient higher-order zig-zag theory for viscoelastic laminated composite plates and solved the problem by the Laplace transform instead of time integration. The effect of viscoelastic interfaces in laminated structures has also been studied for three dimensional static and vibration behavior [12], and cylindrical bending response of piezoelectric laminates with viscoelastic interfaces [13]. Kim et al. [14] studied the residual stresses in thick composite laminates induced during the processing of curing based on a two-dimensional finite element

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model. They used a cure-dependent viscoelastic material model for the analysis. Similarly, Ding et al. [15] studied the process-induced residual stresses via a three dimensional thermo-viscoelastic model. Besides the aforementioned works, Sung [16,17] studied the interlaminar stresses in viscoelastic composites using the finite element method under uniaxial extension, bending and twisting loadings and [18,19] investigated the temperature and moisture effects on viscoelastic responses later.

In the structural design of viscoelastic composite materials, the time-dependent effect must be considered so as to ensure the environmental durability over the entire service life of composite structures. Without taking into the consideration of viscoelastic effect, it cannot predict strain, stress and displacement accurately for viscoelastic materials. In this paper, we report a simple and efficient stress function based approach [20–22] to study the time-dependent stress variations in viscoelastic laminates. To simplify the problem, we adopt the Maxwell model to express the viscoelastic behavior and to study the relaxation effect on interlaminar and free edge stresses in viscoelastic laminates, where the time-dependent relaxation stiffness is in the exponential form and it is easy to implement mathematically. In addition, the proposed methodology prefers using a stress function based theory, where the Lekhnitskii stress functions [23] under the plane strain state are employed and separated by in-plane and out-of-plane stress functions. Although the stress function based approach fails in reflecting the displacement information, it is computationally much more efficient than the commonly used displacement based approach in solving the local stress problem and it can satisfy the pointwise equilibrium equations and the prescribed traction free boundary conditions as well. By taking the principle of complementary virtual work, the coupled fourth-order differential equations are obtained. The solution procedure is firstly by constructing a general eigenproblem. The eigenvalues and eigenvectors compose the general solutions of the fourth-order differential equations. Since we attempt to study the time-dependent relaxation behavior, the constant strain load condition is considered and the results at different times are present. The verification of the proposed method is discussed by comparing the results of three-dimensional finite element analysis. Numerical examples of various layup stacking sequences are also presented as demonstrations to study the time-dependent stress variations. The results will demonstrate the feasibility of using the stress function based approach to study the time-dependent stress relaxations in viscoelastic composite laminates.

2. Formulations

A rectangular viscoelastic laminate with its geometry and coordinate system is shown in Fig. 1, which consists of uniform thickness orthotropic layers. Without considering the temperature and

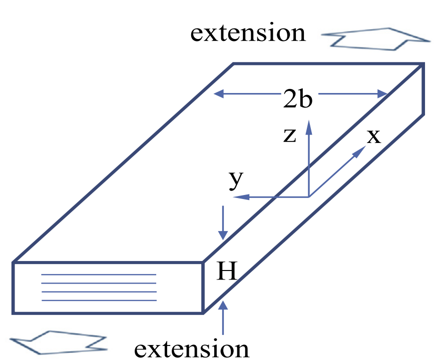

Fig. 1. Coordinate system and geometry of viscoelastic laminate.

moisture effect, the linear viscoelastic behavior can be described by the following Boltzmann supposition integral equations for creeping and relaxation, respectively.

$$\begin{aligned}\{\sigma(t)\} &= [Q_0]\{\varepsilon(t)\} - \textstyle\int_{0+}^{t}\{\varepsilon(\tau)\}[\dot{Q}(t-\tau)]d\tau \\ \{\varepsilon(t)\} &= [J_0]\{\sigma(t)\} - \textstyle\int_{0+}^{t}\{\sigma(\tau)\}[\dot{J}(t-\tau)]d\tau\end{aligned} \tag{1}$$

where t is the time, τ is the time variable of integration, $\sigma(t)$ and $\varepsilon(t)$ are the time-dependent stress and strain, respectively. The matrices $[Q(t)]$ and $[J(t)]$ are the time-dependent stiffness and compliance matrices, and $[Q_0]$ and $[J_0]$ are the initial stiffness and compliance matrices.

In the present study, since we are going to investigate the relaxation of free edge stresses in viscoelastic laminates and to make the problem easier, we choose the relaxation modulus of the Maxwell model which is one of the simplest models for viscoelastic materials and can be expressed by the following time-dependent form without considering the effects of temperature and moisture.

$$[Q(t)] = [Q_0]e^{-\alpha_M t} \tag{2}$$

where α_M is the relaxation parameter normally determined by the experimental relaxation curves.

The developed modeling is based on a constant uniaxial strain load, so that the integration of the relaxation equation can be calculated directly by substituting Eq. (2) into Eq. (1) and simplifying the equation which becomes

$$\{\sigma(t)\} = [Q_0]\{\varepsilon\} - \{\varepsilon\}[Q_0]\int_{0+}^{t}[\alpha_M e^{-\alpha_M(t-\tau)}]d\tau = [Q_0]\{\varepsilon\}e^{-\alpha_M t} \tag{3}$$

Note that Eq. (3) is the time-dependent relaxation equation for a viscoelastic laminae based on the Maxwell model. For orthotropic materials, the stiffness matrix has 9 independent components in their material coordinates. While considering the general layup stacking sequence of laminate, the generalized stiffness matrix has the following relation in terms of the coordinate transform matrix Φ.

$$[\bar{Q}] = [\Phi]^{-1}[Q_0][\Phi]^{-T} \tag{4}$$

Therefore, the time-dependent relaxation equation becomes

$$\{\sigma\} = [\bar{Q}]\{\varepsilon\}e^{-\alpha_M t} \tag{5}$$

The strain vector can be calculated by left multiplying the generalized compliance matrix $\bar{S}$ as follows.

$$\{\varepsilon\} = [\bar{S}]\{\sigma\}e^{\alpha_M t} \tag{6}$$

In the above equation, the stress component σ_1 can be calculated and expressed in terms of ε_1, material coefficients and other five stress components.

$$\sigma_1 = (\varepsilon_1 e^{-\alpha_M t} - \bar{S}_{1j}\sigma_j)/\bar{S}_{11}, \quad (j = 2, 3, \ldots, 6) \tag{7}$$

By substituting Eq. (7) into Eq. (6), all strains can be obtained except ε_1.

$$\varepsilon_i = (\bar{S}_{ij} - \bar{S}_{i1}\bar{S}_{1j}/\bar{S}_{11})\sigma_j + \bar{S}_{i1}/\bar{S}_{11}\varepsilon_1 e^{-\alpha_M t}, \quad (i,j = 2, 3, \ldots, 6) \tag{8}$$

In the present work, the governing equations can be obtained by taking the following principle of complementary virtual work considering the plane strain state.

$$\partial U = \iint \varepsilon_i \delta\sigma_i dydz = 0, \quad (i = 2, 3, \ldots, 6) \tag{9}$$

Substituting Eq. (8) into Eq. (9), it results in the following equation.

$$\begin{aligned}\partial U &= \iint [\delta_j(\bar{S}_{ij} - \bar{S}_{i1}\bar{S}_{1j}/\bar{S}_{11})\delta\sigma_i + (\bar{S}_{i1}/\bar{S}_{11}\varepsilon_1 e^{-\alpha_m t})\delta\sigma_i]d\xi d\eta \\ &= 0 \quad (i,j = 2, 3, \ldots, 6)\end{aligned} \tag{10}$$