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Interlaminar stresses in corrugated laminates

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ABSTRACT

In axially loaded corrugated laminates consisting of more than one layer interlaminar stresses occur due to the curvature which potentially cause delamination of the laminate. Therefore, it is crucial to know how geometry and lay-up influence the interlaminar stress and hence the risk of delamination. In this paper, a parameter study is presented that studies the influence of corrugation amplitude and different lay-ups on interlaminar stresses. We considered two geometries, one consists of circular sections and the other is a sinusoidal shape. From the parameter study we can derive favorable configurations to minimize normalized interlaminar stresses. A numerical model is used to calculate the stress distribution in the cross-section of the corrugation. It considers a generalized plane strain state and uses a unit-cell approach. The model is validated with experiments in a tensile test using digital image correlation (DIC).

1. Introduction

The geometry effects of singly-curved corrugated sheets create extreme anisotropy. As other authors [1,2] have suggested, corrugated sheets are ideal candidates for the design of flexible skins as a structural element in morphing-wing design, where high stretch ability along the chord direction must be combined with high contributions to structural stiffness and strength along the span direction of an airplane wing [3]. Other authors [4,5] investigated the potential of morphing wing concepts to achieve higher control and flight performance at lower weight than conventional solutions. To achieve higher mass-specific stiffness and strength and higher limit strain, the base sheet should be a laminate made from carbon-fiber reinforced plastic (CFRP). A recent review article [6] summarizes the state-of-the-art of the research investigating corrugated sheet concepts emphasizing the relevance of these structures.

Flexible skins must not impair the aerodynamic properties of the morphing wings. It has been found that the aerodynamic behavior improves with decreasing corrugation amplitudes and periodic cell lengths [5,7]. Smaller periodic lengths lead to higher laminate thickness-to-curvature ratios, which increase the local interlaminar stresses caused by the corrugation geometry. The interlaminar stresses can cause layer debonding at relatively low external loads [8]. The present work investigates the influence of corrugation parameters on the ratio of out-of-plane to in-plane stress components in order to identify, in Section 4.3, design limits with respect to interlaminar strength. We focus on periodic corrugated structures, however, the considerations are in general also applicable for curved laminates.

The analysis of corrugated structures made from anisotropic material can lead to enormous computational costs. Conventional finite element methods are not efficient if the structure contains many periods of the corrugations and consists of many layers. Dayyani et al. presented a detailed nonlinear finite element study of corrugated structures [9], but this would be too time consuming for our purposes since we aim to perform parameter studies. Therefore, fast models are needed to calculate the mechanical response. Several analytical models exist to analyze the initial stiffness corrugated structures. Xia et al. suggested a homogenization model to calculate the stiffness matrix for thin balanced laminates, thus ignoring the coupling stiffness matrix B [10,11]. Winkler et al. [12] proposed an equivalent model for circular sections. Mohammadi et al. suggested an analytical model for trapezoidal shapes based on a homogenization approach [13]. These models are only valid for thin laminates and therefore are note appropriate for the present study where we investigate relatively thick laminates. Hence, a numerical approach is more suitable for the present work. Peng et al. [14] introduced a mesh-free Galerkin model to calculate the elastic stiffness behavior of trapezoidal and sinusoidal shaped corrugations. The model was extended by Liew et al. [15] to simulate the nonlinear response of corrugated laminates.

Various models exist to calculate the stress distribution in curved structures. An analytical model to predict the stress in curved laminates was suggested by Roos et al. and Kress et al. respectively [16,17]. Fraternali et al. proposed a one-dimensional

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finite element model to calculate interlaminar stresses in curved beams that are loaded in three-point bending [18]. Shenoi et al. presented an analytical model based on elasticity-theory to predict through-thickness stresses in curved laminates and sandwich panels [19]. Gonzalez-Cantero et al. introduced a semi-analytic method to predict interlaminar stresses in curved beams with constant curvature [20]. All these models are not specifically developed to analyze periodic corrugated structures, are not suitable for arbitrary shapes, do only consider certain load cases or are restricted to single components of the interlaminar stress.

Recently, Kress et al. suggested a numerically efficient model using a unit-cell approach and a generalized plane strain state to model corrugations of any shape, thickness and with arbitrary lay-ups [21–23]. The model is suitable for all load cases describing the full mechanical response of the structure. This model is applied and extended in this paper in order to analyze the spatial stress distribution in corrugated laminate cross-sections.

The objectives of this paper are to investigate the influence of geometry and laminate lay-up onto interlaminar stresses. We further ask the question whether we can identify favorable geometries in order to minimize the interlaminar stresses. We compare a sinusoidal shaped corrugation to a corrugation consisting of circular sections. We consider four different lay-ups consisting of pure twill or UD layers and combinations of them. A parameter study is presented where we investigate the influence of the corrugation amplitude and the lay-up on the mechanical behavior, namely the interlaminar stresses and the axial stiffness, for the different geometries. Further, we present in this paper an experimental validation of the used numerical model where we measured displacements and strains.

The following section of the paper introduces the numerical model, geometry definition and load cases. Then the experiments are described and the experimental results are presented and discussed. The next section describes the parameter study and shows and discusses the numerical results of the simulations, including the stress distribution for certain examples and the results of the parameter study. The paper closes with a conclusion of the present work.

2. Numerical model

We used a numerical model that is able to calculate the mechanical response of corrugated laminates with arbitrary thickness and lay-up. Our model is based on the FEM model that was suggested by Kress and Winkler [21–23]. The model uses a unit-cell approach and a generalized plane strain state to reduce the computational costs. The generalized plane strain state assumes that the stresses and strains do not change with respect to the out-of-plane x direction. It is derived from a simplified mechanical equilibrium:

$$\begin{aligned} \tau_{yx,y} + \tau_{zx,z} &= 0 \\ \sigma_{y,y} + \tau_{zy,z} &= 0 \\ \tau_{yz,y} + \sigma_{z,z} &= 0 \end{aligned} \tag{1}$$

Uniform strains $\hat{\epsilon}_x^0$, uniform bending about y $\hat{\epsilon}_x^1$ and twist $\hat{\epsilon}_{xy}^1$ are used to force deformations. They are compatible with the generalized-plane-strain assumption and they lead to a displacement field of the form:

$$\begin{aligned} u_x &= u_x(y,z) + x\left(\hat{\epsilon}_x^0 + z\hat{\epsilon}_x^1\right) + \frac{1}{2}yz\hat{\epsilon}_{xy}^1 \\ u_y &= u_y(y,z) + \frac{1}{2}zx\hat{\epsilon}_{xy}^1 \\ u_z &= u_z(y,z) - \frac{1}{2}x^2\hat{\epsilon}_x^1 - \frac{1}{2}xy\hat{\epsilon}_{xy}^1 \end{aligned} \tag{2}$$

This displacement solution consists of an inner FE solution $u_i(y,z)$ and the prescribed strains $\hat{\epsilon}_i^{0,1}$. The inner FE solution is not a function of the transverse direction x. Hence, it is sufficient to calculate the FE solution at one cross-section position only. This reduces the computational costs.

The strains are calculated from the displacement solution using the linearized kinematic relations,

$$\boldsymbol{\epsilon} = \boldsymbol{L}\boldsymbol{u} + \hat{\boldsymbol{\epsilon}} \tag{3}$$

where the linear differential operator $\boldsymbol{L}$

$$\boldsymbol{L}^T = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial}{\partial y} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix} \tag{4}$$

is adapted to the plane-strain assumption as it contains no derivatives with respect to x and $\hat{\boldsymbol{\epsilon}}$ contains the prescribed macro strains:

$$\hat{\boldsymbol{\epsilon}}^T = [\hat{\epsilon}_x^0 + \hat{\kappa}_x z \; 0 \; 0 \; 0 \; 0 \; \hat{\kappa}_{xy} z]. \tag{5}$$

The stresses are calculated from the strains using the generalized Hook's law:

$$\boldsymbol{\sigma}_{xyz} = \boldsymbol{C}\boldsymbol{\epsilon} \tag{6}$$

where $\boldsymbol{\sigma}_{xyz}$ and $\boldsymbol{\epsilon}$ denote the stress and strain vector, respectively. $\boldsymbol{C}$ represent the material law for orthotropic materials in global coordinates.

The above expressions for strains and stresses are used to formulate the weak variational form

$$\int_\Omega \delta\boldsymbol{u}^T\boldsymbol{L}^T\boldsymbol{C}(\boldsymbol{L}\boldsymbol{u} + \hat{\boldsymbol{\epsilon}})d\Omega - \int_\Gamma \delta\boldsymbol{u}^T\boldsymbol{\Phi}\hat{\sigma}d\Gamma = 0 \tag{7}$$

which is transformed by the finite-element method to a numerical system of equations where the unknown mesh-node displacements $\tilde{\boldsymbol{u}}$ respond to the forces contained in the right-hand-side $\boldsymbol{r}$ and the structural properties are mapped with the stiffness matrix $\boldsymbol{K}$:

$$\boldsymbol{K}\tilde{\boldsymbol{u}} = \boldsymbol{r}. \tag{8}$$

The standard notation of the stiffness matrix K is

$$\boldsymbol{K} = \sum_{k=1}^{N_{el}} \int_{\Omega_{el}} \boldsymbol{B}^T\boldsymbol{C}\boldsymbol{B}d\Omega \tag{9}$$

where N_{el} is the number of finite elements in the mesh, Ω_{el} is element domain, $\boldsymbol{B}$ relates node displacements to element strain distributions,

$$\boldsymbol{\epsilon} = \boldsymbol{B}\tilde{\boldsymbol{u}} = \boldsymbol{L}\boldsymbol{\Phi}\tilde{\boldsymbol{u}} \tag{10}$$

where $\boldsymbol{\Phi}$ are the element displacement approximation functions. The right-hand side reflects natural boundary conditions and the macro strains to be described for the generalized plane-strain condition,

$$r = \sum_{k=1}^{N_{el}} \int_\Gamma \boldsymbol{\Phi}^T\hat{\boldsymbol{\sigma}}d\Gamma - \sum_{k=1}^{N_{el}} \int_{\Omega_{el}} \boldsymbol{B}^T\boldsymbol{C}\hat{\epsilon}d\Omega \tag{11}$$

where $\hat{\boldsymbol{\sigma}}$ contains the tractions specified on the boundary Γ.

The form allows for a planar mesh where each node carries three displacement degrees-of-freedom. By using a unit-cell, we assume that the corrugation pattern is periodic and we apply periodic boundary conditions on both ends.

To evaluate the stresses and strains at an optimal location in the finite element, the Barlow points are well known and used as a standard procedure [24]. In our case the Barlow point coincide with the three times three Gauss points. Using the constitutive law, we can calculate the stresses from the strains. We define the stresses in the corrugated laminate according to Fig. 1 where