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The Certain Generalized Stresses Method for static analysis of multilayered composite plates with variability of material and physical properties

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ABSTRACT

In this paper, the Certain Generalized Stresses Method (CGSM) is developed for static probabilistic analysis of multilayered composite plates modeled by finite elements. The material and the physical properties are considered as random parameters. The variability of a displacement can be evaluated by a Monte Carlo Simulation (MCS) using an explicit expression. For calculating the variability of a displacement, only one nominal finite element analysis with two load cases is required. The variability of strains, stresses and failure criteria can also be evaluated by using the CGSM. Two examples, a two-layer bending plate and an eight-layer bending plate, are studied. The results are compared with those obtained by the direct MCS, considered as a reference, and those presented in the literature. The comparison shows that the CGSM + MCS approach provides quite accurate results and highlights the high computational efficiency of the proposed method.

1. Introduction

Because of their good mechanical strength and low density, composite laminated structures are widely used in various industrial sectors like construction, aerospace, transport and so on. But their manufacturing process induces a higher variability of properties than that of conventional homogeneous and isotropic structures.

So far, many studies have been investigated to take into account the uncertainties of the material or physical properties of composite structures in numerical models. Several methods have been developed to study the homogenization and estimation of properties of composites. Sakata et al. use a perturbation method [1] and a Kriging method [2] to study the homogenization of the elastic properties. Dwaikat et al. [3] study the effect of the stochastic nature of the constituent parameters on the elastic properties of fibrous nano-composites with the direct Monte Carlo Simulation (MCS), which runs a great number of simulations with random trials. Kamiński and Leśniak [4] study the homogenization of metallic fiber-reinforced composites using the perturbation method.

For membrane composite structures, there are fewer studies. Van Vinckenroy and de Wilde [5] use the direct MCS to evaluate the variability of the stresses of a perforated plate. The variability of the strains of a rectangular panel is investigated by Ngah and Young [6] using a spectral method. The influence of uncertain Poisson's ratio on the out-of-plane behavior of composite plates is studied by Noh and Seo [7] using a weighted integral method.

Bending composite structures have drawn more attention. Park et al. [8] study the variability of displacements and stresses with a perturbation method. A spectral method is employed by Chen and Soares [9] to evaluate the variability of the displacements of a plate under a concentrated load. António and Hoffbauer [10] use a perturbation method for the probabilistic analysis of the displacements and the failure criteria of a composite shell. A weighted integral method is used to study the influence of Poisson's ratio by Noh and Park [11] as well as that of elastic and shear moduli by Noh [12].

There are also many studies about the reliability of composite structures. Jeong and Shenoi [13], Lin [14], Frangopol and Recek [15], employ the direct MCS to predict the reliability of composite structures with uncertain parameters. The perturbation method is respectively used by Salim et al. [16], Onkar et al. [17,18], and Lal et al. [19] for the probabilistic failure analysis of laminated composite plates.

However, these existing methods, such as the direct MCS, are either very costly in computation time or not accurate for a high-level variability. So developing an economical and reliable method

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accounting for the variability in the finite element calculations is a very interesting challenge. The Certain Generalized Stresses Method (CGSM) is firstly proposed by Lardeur et al. [20–23] for the probabilistic analysis of isotropic bars and beams. Mahjudin et al. [24] develop this method for homogenous and isotropic plates. This method can efficiently provide accurate results, even if the level of variability and the number of variables are large. Moreover, it is non-intrusive and able to take into account the variability of not only material properties (elastic moduli...) but also physical properties (plate thickness...).

In this article, the CGSM is developed for multilayered composite plates with uncertain material and physical properties. These uncertain parameters are represented by random fields which are discretized by the Karhunen–Loève expansion and two approaches: the midpoint method and the local average method. The CGSM is based on the assumption that the generalized stresses are independent of uncertain parameters. Thanks to this assumption, the internal strain energy can be expressed for any combination of uncertain parameters. Using Castigliano's theorem, an explicit expression of displacement can be obtained. Then a MCS is performed on this expression, which allows to calculate the statistical quantities (mean value, standard variation, distribution...) of a displacement. This method can also be employed to evaluate the variability of strains, stresses and failure criteria.

In the next section, the Karhunen–Loève expansion and two approaches, the midpoint method and the local average method, are presented. Section 3 gives a short description of the mechanical behavior of multilayered composite structures. In Section 4, the CGSM formulations for composite plates are presented. In Section 5, two numerical examples are studied with various correlation lengths and input variability levels. For verifying the numerical accuracy and the computational efficiency of the CGSM, the results are compared with those obtained by the direct MCS and those presented in the literature. Conclusions are given in the last section.

2. Random field

For a parametric approach, the uncertain parameters are represented by random variables or random fields. The variability of a composite structure modeled by finite elements is studied by using random fields in this paper. In order to obtain a random field, first of all, a covariance matrix $[Cov]$ should be calculated. Two methods are used here to construct $[Cov]$: the midpoint method and the local average method.

2.1. Midpoint method

The midpoint method is the most widely used method because of its simplicity. In this method, the random fields are calculated at the center of each element. The covariance matrix is defined by:

$$[Cov]_{ij} = \sigma^2 \exp\left(-\frac{|\Delta x_{ij}|}{\lambda_x} - \frac{|\Delta y_{ij}|}{\lambda_y}\right) \tag{1}$$

where σ is the standard deviation of the uncertain parameter, Δx_{ij} and Δy_{ij} are respectively the distances between the midpoints of the elements i and j in the x and y direction, λ_x and λ_y are respectively the correlation lengths in the x and y direction.

2.2. Local average method

The limit of the midpoint method is that the variability of the response cannot be well estimated when the correlation length is small or the mesh is coarse. In this case, the local average method is employed. It was proposed by Vanmarcke and Grigoriu [25] and developed by Zhu et al. [26,27]. In this method, the random fields are obtained by an integration over the elements, rather than the distance between the element centers. The covariance matrix between two elements is thus described:

$$[Cov]_{ij} = \frac{\sigma^2}{A_i A_j}\int_{A_i}\int_{A_j} \exp\left(-\frac{|\Delta x_{ij}|}{\lambda_x} - \frac{|\Delta y_{ij}|}{\lambda_y}\right) dA_i dA_j \tag{2}$$

where A_i and A_j are the areas of elements i and j, Δx_{ij} and Δy_{ij} are respectively the distances between two arbitrary points of the elements i and j in the x and y direction.

According to Ghanem and Spanos [28], the random field can be discretized by the Karhunen–Loève expansion. $\langle E(x,\theta)\rangle$ denotes a random field whose mean value is $\mu(x)$. It can be represented by:

$$\langle E(x,\theta)\rangle = \mu(x) + \sum_{i=1}^{m}\sqrt{\lambda_i} f_i(x)\langle\xi(\theta)\rangle \tag{3}$$

where m is the number of terms retained, λ_i and $f_i(x)$ are respectively the eigenvalues and eigenvectors of the covariance matrix $[Cov]$. $\langle\xi(\theta)\rangle$ is a vector of random variables which is defined by a truncated Gaussian distribution law. The Eq. (3) can be written as:

$$\langle E(x,\theta)\rangle = \mu(x) + [L]\langle\xi(\theta)\rangle \tag{4}$$

with

$$[L] = [Cov]^{1/2} \tag{5}$$

In order to get $[L]$, two methods can be used to decompose the covariance matrix $[Cov]$: the Cholesky decomposition or the singular value decomposition. These two methods generally lead to close results.

3. Mechanical behavior of multilayered composite structures

3.1. Constitutive law

For multilayered composite structures, the mechanical behavior is governed by Hooke's law. Taking into account the assumption $\sigma_3 = 0$ used in classical plate theories exploited in this study, the constitutive relations of each layer in the material coordinate system can be written as:

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{6}$$

$$\{\tau\} = [Q_S]\{\gamma\} \tag{7}$$

where $\{\sigma\}$, $\{\varepsilon\}$, $\{\tau\}$ and $\{\gamma\}$ are respectively plane stress vector, plane strain vector, shear stress vector and shear strain vector. The stiffness matrices $[Q]$ and $[Q_S]$ are defined by:

$$[Q] = \begin{bmatrix} \frac{E_1}{1-\vartheta_{12}\vartheta_{21}} & \frac{\vartheta_{21}E_2}{1-\vartheta_{12}\vartheta_{21}} & 0 \\ \frac{\vartheta_{21}E_2}{1-\vartheta_{12}\vartheta_{21}} & \frac{E_2}{1-\vartheta_{12}\vartheta_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \tag{8}$$

$$[Q_S] = \begin{bmatrix} G_{13} & 0 \\ 0 & G_{23} \end{bmatrix} \tag{9}$$

where E_1 is the longitudinal elastic modulus, E_2 is the transversal elastic modulus, ϑ_{12} and ϑ_{21} are Poisson's ratios, G_{12} is the in-plane shear modulus, G_{13} and G_{23} are the out-of-plane shear moduli.

The direction of fibers in each layer of laminated composite structures may be different, which means a stacking angle θ between material axes and global axes (Fig. 1). Referred to global coordinate system, the constitutive relations can be transformed into:

$$[Q'] = [T_1]^T[Q][T_1] \tag{10}$$