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Failure envelopes for laminated composites by the parametric HFGMC micromechanical framework

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ABSTRACT

Micromechanically doubly periodic parametric High Fidelity Generalized Method of Cells, in conjunctions with continuum damage mechanics considerations, is presented to determine failure envelopes of unidirectional composite materials. The methods is based on an incremental procedure in which the local damage variables and global stresses are monitored during the strain softening to provide the value of the envelopes at which ultimate failure occurs. The micromechanically established failure envelopes are compared with the well known macrolevel based failure surfaces and with experimental data of multiaxial failure stresses found in the literature. It is shown that the new micromechanical failure envelopes are effective in predicting the multi-axis stress failures.

1. Introduction

Failure prediction of composite material is critical for the design of composite materials and structures. One approach to determine the failure values of a composite material is expressed in the macroscale level in which the failure envelopes are defined by polynomials of the stresses and the unidirectional ultimate stresses in the composite level such as Tsai–Wu [1], Hoffman [2], Tsai-Hill [3], Christensen [4]and Hashin [5]. Some of the material failure criteria distinguish between failure in fiber and matrix modes as in Christensen [4], Hashin [5] and Puck [6]. On the other hand, microscale level failure criteria consider the local elastic fields of the fiber and matrix constituents such as strain invariant failure theory (SIFT) [7] and Multi Continuum Theory [8]. More recent criterion uses terms of strain and stress invariants in the expressions for fiber and matrix failure [9].

In the framework of these microscale approaches which are used to predict the ultimate failure values, the local elastic fields can be determined by using the finite element method (e.g. Mayes and Hansen [8], Ha et al. [10], Welsh et al. [11], Montesano and Singh [12], see also the reviews of Mayes [13] and Liu and Zheng [14]). A micromechanical model can be embedded in a finite element model and used to investigate damage mechanism in unidirectional composites [15,16]. Other works suggest the employment of micromechanical methods which predict the elastic field and subsequently evaluate the damage in composite, e.g. Aboudi [17], Haj-Ali [18], Tang and Zhang [19], Ye et al. [20], Haj-Ali and Aboudi[21], Moncada et al. [22] and Naik et al. [23].

In the present article, a micromechanical method is adopted for the prediction of failure envelopes in composites. To this end, the High Fidelity Generalized Method of Cells (HFGMC) micromechanical method is employed. This method is capable to predict the effective mechanical properties and the local elastic field in composites with periodic microstructures. In the framework of this method, a repeating unit cell (RUC) is identified and divided into subcells between which continuity of displacements and tractions are imposed in average sense. Furthermore, the equilibrium equations in the subcell are enforced. In addition, the periodicity conditions are imposed according to which the displacements and tractions at opposing edges of the RUC are required to be equal. For a comprehensive description of this method see Aboudi et al. [24]. Recently, Haj-Ali and Aboudi [25] proposed a new formulation for a parametric doubly and triply periodic HFGMC in which general quadratic or hexahedron subcell can be used by mapping form physical domain to a parent domain. It has been shown that the method is capable of predicting the effective moduli and the elastic field distributions in various types of composites.

In this study, the parametric doubly-periodic HFGMC micromechanical method is proposed for the prediction of failure envelopes in composite materials. To this end, the HFGMC is employed, in conjunction with continuum damage mechanic (CDM) theory

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[26,27], to establish these surfaces. The evolving damage law with applied loading on the composite given in Lemaitre and Desmorat [26] is modified in order to make it insensitive to the sign of the shear stress. This law is implemented in every subcell which allows to follow the evolution and distribution of the damage in the RUC. By monitoring the resulting global stresses, the ultimate stress values are determined and used to generate the requested failure envelopes of the composite. Comparisons of the resulting micromechanically based failure envelopes with models based on macroscale approaches and experimental results are presented.

This paper is organized as follows. In Section 2, the doubly-periodic parametric HFGMC formulation is outlined. In Section 3, the modification of the damage law and its implementation is described. The generation of failure envelopes by the micromechanical HFGMC and analytical formulation for these envelopes are given in Section 4. Comparisons with macroscale approach and experimental results are presented in this section. Finally, in Section 5, conclusion and future work are proposed.

2. Parametric HFGMC

In this section, the governing equations of doubly periodic parametric HFGMC are discussed. Effective mechanical properties, which are typical HFGMC output, are compared with experimental data. Representative results of stress distribution map in a repeating unit cell (RUC) of unidirectional composite are shown. However, damage implementation will be discussed in Section 3.

Consider a doubly periodic composite described with respect to global coordinates x_2 and x_3, see Fig. 1(a). An RUC is identified which is described with respect to local coordinates y_2 and y_3 as shown in Fig. 1(b). The RUC is discretized into arbitrary number of subcells as illustrated in Fig. 1(c). The micromechanical analysis of triply periodic composites has been presented by Haj-Ali and Aboudi [25]. Presently, the corresponding analysis for doubly periodic composites is summarized in the following. A characteristic subcell is shown in Fig. 1(d). The parametric HFGMC is formulated in the parent (r, s) domain, Fig. 1(e), in which a subcell is represented by a square shape with boundaries located between $-1 \leqslant r, s \leqslant 1$. The transformation from the parent domain (r, s) to the physical domain (y_2, y_3) is determine by the well known linear mapping

$$y_i(r,s) = \sum_{k=1}^{4} H_k(r,s) y_i^{(k)} \tag{1}$$

$$H_1 = \frac{1}{4}(1-r)(1-s), \quad H_2 = \frac{1}{4}(1+r)(1-s)$$
$$H_3 = \frac{1}{4}(1+r)(1+s), \quad H_4 = \frac{1}{4}(1-r)(1+s)$$

where k indicates a corner of a subcell, r and s are the uniform parametric coordinate system of the parent domain and $y_i^{(k)}$ is the coordinate of the kth corner in the physical domain. The Jacobian is defined by:

$$J = \begin{vmatrix} \frac{\partial y_2}{\partial r} & \frac{\partial y_3}{\partial r} \\ \frac{\partial y_2}{\partial s} & \frac{\partial y_3}{\partial s} \end{vmatrix} \tag{2}$$

In every subcell, a three-dimensional displacement polynomial is defined as follows [25]

$$\begin{aligned} \mathbf{u} = {} & \epsilon^0 \cdot \mathbf{x} + \mathbf{W}_0 + \frac{1}{2}(\mathbf{W}_2 - \mathbf{W}_4)r + \frac{1}{2}(\mathbf{W}_3 - \mathbf{W}_1)s \\ & + \frac{1}{4}(\mathbf{W}_2 + \mathbf{W}_4 - 2\mathbf{W}_0)(3r^2 + rs - 1) \\ & + \frac{1}{4}(\mathbf{W}_1 + \mathbf{W}_3 - 2\mathbf{W}_0)(3s^2 + rs - 1) \end{aligned} \tag{3}$$

where $\mathbf{W}_i$ are micromechanical unknowns to be determined in the following. In this equation, the expression for the displacement $\epsilon^0 \cdot \mathbf{x}$ includes the remote strain field applied on the composite, $\mathbf{W}_0$ is associated with a rigid body motion, and $\mathbf{W}_i$ $(i = 1, 2, 3, 4)$ are the average displacement of the ith subcell face. It should be

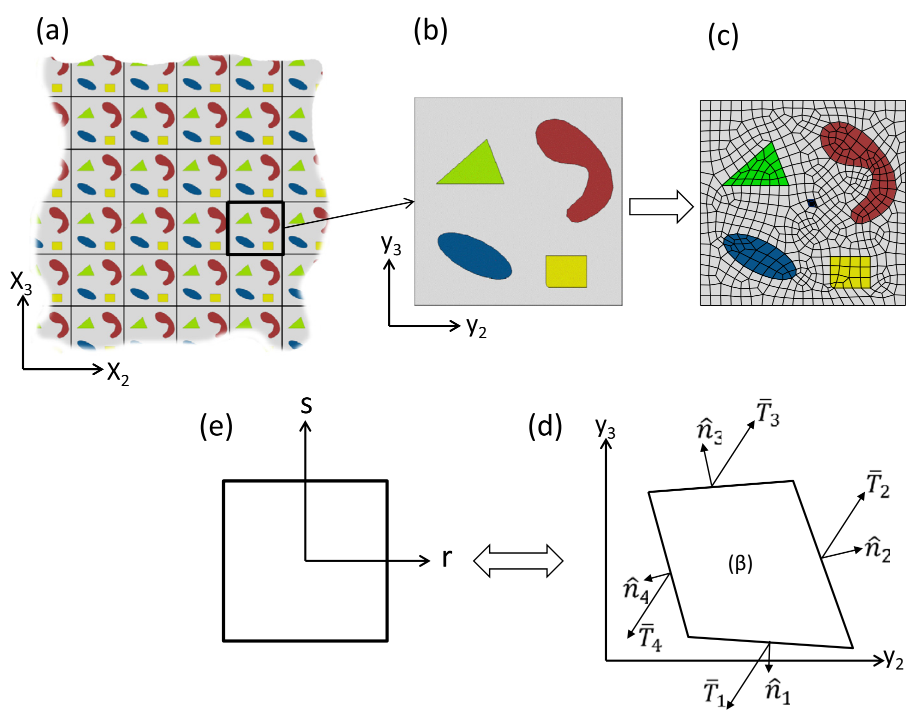

Fig. 1. Schematic representation of doubly periodic material at the composite and subcell level. For a given periodic composite material (a), a RUC can be identified and isolated (b). Subfigure (c) shows a RUC divided into subcells. The mapping of subcell from the physical to the parametrical coordinate system is illustrated in subfigure (d) and (e). $\bar{\boldsymbol{T}}_i$ and $\hat{n}_i$ are the traction applied on face i and its normal vector, respectively.