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Stochastic thermal buckling analysis of laminated plates using perturbation technique

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ABSTRACT

Composites are known to display a considerable amount of scatter in their material properties due to large number of parameters involved with the manufacturing and fabrication processes. This paper is concerned with the effect of random system properties on critical thermal buckling temperature of composite laminated plates with temperature dependent properties using micromechanical approach. System parameters are assumed as independent random variables. In this analysis, based on the classical lamination theory in conjunction with the Hamilton's principle, the basic formulation of random eigenvalue problem has been deduced. A mean-centered first order perturbation technique is used to compute the second-order statistics (mean and standard deviation) of the critical thermal buckling temperature. The performance of outlined stochastic approach has been validated by comparing the present results with those available in the literature and independent Monte Carlo simulation. The effect of random material properties, thermal expansion coefficients, fiber volume fractions, aspect ratios, laying angels and boundary conditions on the critical thermal buckling temperature are presented.

1. Introduction

Fiber reinforced composite materials has been developed over the last three decades and used for a great variety of engineering applications including aircraft, submarine, automotive, naval and space structures. These structures are often subjected to severe thermal environments and the thermal loads become a primary design factor in specific cases. The structures made of composite materials have more randomness and variability in the system properties compared to conventional isotropic structures as a large number of parameters are involved with their fabrication and manufacturing processes. Some variations of system parameters, such as material properties, thermal expansion coefficients, fiber volume fractions and laying angels, are inevitable even for the composite laminates manufactured carefully in a laboratory. Uncertainties in these lead to uncertainties in the response behavior of the structure. In many sensitive applications accurate knowledge of the response is very important and at times may be critical. In deterministic analysis, the system uncertainties are neglected and only the mean response is given which obviously misses the deviation caused by the randomness in the system parameters. It is, therefore, necessary that the material properties, thermal expansion coefficients, fiber volume fractions and laying angels, an important set of system variables, be modeled as random variables (RVs) to achieve an improved accuracy of the analysis.

Considerable research has been done to characterize the thermal buckling response of the structures made of composites based on deterministic analysis [1–7]. Relatively little efforts have been made by the researchers and investigators on the prediction of the thermal buckling response of the structures made of composites with random system parameters using micromechanical approach, notable among them are Lal [8–10] and Talha and Singh [11]. However, no work, dealing with thermal buckling of the laminated composite plate with random material properties, random thermal expansion coefficients, random fiber volume fractions and random laying angels using micromechanical approach, is reported in the literature to the best of the author' s knowledge.

The study about the effects of randomness in system properties on free vibration and elastic buckling of laminated composite plate has received much attention. Onkar and Yadav [12,13] have analyzed the effect of material parameter dispersion on the large amplitude free vibration of especially orthotropic laminated composite plates using the classical laminate theory and Von-Karman non-linear strain–displacement relation based first order perturbation technique (FOPT). The mean and standard deviations of the

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first five natural frequencies for laminated rectangular plates have been worked out by Singh et al. [14], based on higher order shear deformation theory (HSDT) using the FOPT. Chen et al. [15] presented a probabilistic method to evaluate the effect of uncertainties in geometrical and material properties of structure on the random vibration response. Nakagiri et al. [16] studied simply supported (SS) laminated plate with a stochastic finite element method (SFEM) taking fiber orientation, layer thickness and number of layers as random variables, and found that the overall stiffness of fiber reinforced plastic laminated plates is found out to be largely dependent on the fiber orientation. Englested and Reddy [17] studied metal matrix composites based on probabilistic micromechanics nonlinear analysis. They used Monte Carlo simulation (MCS) with different probabilistic distributions to incorporate the uncertainty in basic material properties. Graham and Siragy [18] have studied the variability of the random buckling loads of beams and plates with stochastically varying material and geometric properties using the concept of the variability response function. Lin [19] investigated the buckling failure probability evaluation of laminated composite plates subjected to different in-plane random loads using the SFEM, the feasibility and accuracy of the result are validated using the results obtained by the MCS. Besides, using different methods, the reliability predictions of laminated composite plates with random system parameters subjected to transverse loads are also performed by Lin [20]. Stefanou and Papadrakakis [21] computed response variability for the case of combined uncertain material (Young's modulus, Poisson's ratio) and geometric (thickness) properties using the SFEM. Most recently, the effect of random system properties, plate geometry, stacking sequences, support conditions and fiber volume fractions on hygrothermomechanical buckling load of the laminated shells and panels are presented using C^0 FEM in conjunction with FOPT based on the HSDT [22,23].

This paper investigates the thermal buckling characteristics of laminated composite plates in the presence of small random variability of system properties using the classical laminate theory in conjunction with the Hamilton's principle. The fiber elastic modulus, fiber shear modulus, fiber Poisson ratio, matrix elastic modulus, matrix shear modulus, matrix Poisson ratio, fiber volume fraction, fiber thermal expansion coefficients, matrix thermal expansion coefficient and laying angels of the plate are modeled as independent random variables (RVs), respectively. The first order perturbation technique is employed to determine the second order statistics (mean and standard deviation) of the critical thermal buckling temperature of laminated composite plates with and without temperature dependent properties. Furthermore, the effect of different aspect ratios and boundary conditions on the critical thermal buckling temperature are also examined. All of the numerical results obtained by the present solution approach (FOPT) are validated with those obtained by independent MCS.

2. Thermal buckling problem formulation of composite laminated plate

2.1. Displacement field model

A symmetric laminated composite panel is considered with the length a, width b and thickness h, respectively, as shown in Fig. 1. The origin of the coordinate system (x, y, z) is located on the reference plane in such a way that $x = -a/2$, $a/2$ and $y = -b/2$, $b/2$ indicate the edges of the laminated panel and the $z = -h/2$ and $h/2$ indicate the upper and lower surfaces.

Using the classical laminate theory, the nonlinear strain–displacement relations of a material point located at (x, y, z) in the panel may be written as follows

$$\{\varepsilon\} = \{\varepsilon_m\} + z\{\kappa\} \tag{1}$$

where ε_m and κ are the membrane strain vector and the curvature changes vector. These components can be written by the vibratory displacements as

$$\{\varepsilon_m\}^T = \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{\partial y}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \frac{\partial y}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \end{Bmatrix} \quad \{\kappa\}^T = \begin{Bmatrix} -\frac{\partial^2 w}{\partial y^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial xy} \end{Bmatrix} \tag{2}$$

where u, v and w are the displacement components in the Cartesian coordinate system x, y and z.

2.2. Stress–strain relation

The constitutive law of thermo-elasticity for the materials under consideration which relates the stress with strain for the kth lamina oriented as an arbitrary angle with respect to the reference axis for an orthotropic layer is given by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} - \alpha_x \Delta T \\ \varepsilon_{yy} - \alpha_y \Delta T \\ \varepsilon_{xy} - \alpha_{xy} \Delta T \end{Bmatrix}_k \tag{3}$$

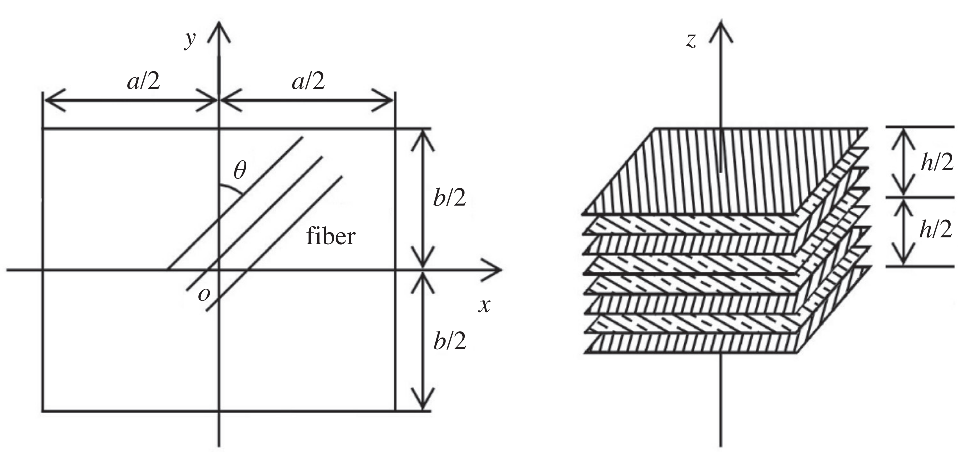

Fig. 1. The configuration of the fiber reinforced laminated plate structure.