



# Analytical homogenization for in-plane shear, torsion and transverse shear of honeycomb core with skin and thickness effects



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## ABSTRACT

The homogenization of the honeycomb sandwich core enables to obtain an equivalent homogeneous solid and its elastic moduli thus to make numerical modeling very efficient. In the present paper, the in-plane shear, torsion, in plane-shear coupled with torsion and transverse shear are studied with skin effect: the honeycomb core's deformation is constrained by two fairly rigid skins, so the core's stiffness is largely increased. An analytic homogenization method, using trigonometric function series is proposed to study the influence of the honeycomb thickness on the elastic properties, and the upper and lower bounds of their equivalent elastic moduli can be given directly by the analytical homogenization models (H-models). A very good agreement has been achieved between numerical results issued from the H-models and 3D FE modeling.

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## 1. Introduction

Honeycomb cellular materials are widely used in lightweight engineering applications, such as in aerospace, railway and automotive industries. These materials are commonly used in sandwich panels, consisting in two relatively thin, high-density and high-strength face sheets, adhesively-bonded to a relatively thicker, softer and low-density core. In such composite structures, laminates and honeycomb play different roles. Face sheets carry the in-plane and bending loads whereas the core maintains the distance between the sheets and carries the out-of-plane pressure loads and shear loads on the sandwich structure. Face skins and honeycomb core may be made of various materials, including aluminum [1], stainless steel, fiber-reinforced polymers and paperboard.

Due to their high geometrical and structural complexity, a realistic modeling of the mechanical behavior of honeycomb sandwich structures requires large numerical models, involving many cells and thus resulting in time consuming calculations. In practice, for numerical efficiency, the analysis of such materials during the design process relies on effective properties issued from a homogenization approach, rather than those issued by considering the real cellular structure. Indeed, several methods can be used to derive equivalent effective properties of periodic or nearly-periodic

non-homogeneous media such as honeycomb sandwich structures (see [2] for an overview).

Among the pioneering studies, one can mention the work by Kelsey et al. [3] and Chang and Ebcioğlu [4], who determined the effective shear moduli of hexagonal honeycomb sandwich cores using direct stress and strain redistributions along the boundaries of a representative volume element (RVE). Gibson and Ashby [5] have provided the concept of a representative unit cell to derive the effective overall core properties under macroscopically homogeneous loading conditions. Within a rigorous mechanical and mathematical framework, Bishop and Hill [6] have determined effective properties using energetic considerations, whereas Hashin and Shtrikman [7] used variational approaches. Using an analytical approach, Meraghni et al. [8] proposed an analysis of the transverse shear modulus of honeycomb cores based on the modified laminate theory.

In the present study, an energetic homogenization procedure has been developed and applied to a hexagonal honeycomb sandwich structure, taking into account both core's thickness and skin effects. In such approach, it is assumed that the effective strain energy stored in a representative volume element (RVE) of the given structure and an equivalent volume element of the effective homogeneous medium have to be equal if both RVE are submitted to the same macroscopically equivalent loading conditions. To our knowledge, few authors investigated both core's thickness and skin effects together. One can mention the work by Grediac [9], who investigated the influence of the core height on the transverse shear modulus. Using the finite element method, he has presented

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a method for calculating the transverse shear modulus of honeycomb cores as well as the stress distribution in the honeycomb walls. He pointed out that the transverse shear modulus decreases as the core's thickness increases. Later, Pan et al. [10] have investigated the transverse shear mechanical behavior and failure mechanism of aluminum alloy honeycomb cores. The authors have shown that in order to predict accurately the equivalent transverse shear modulus and strength, not only shear deformation but also bending deformation of cell walls should be considered. By application of the cantilever beam theory and thin plate shear buckling theory in conjunction with simplifying assumption as to the displacement in the cores, they concluded that the contribution of bending deformation of cell walls to equivalent transverse shear modulus is obvious with decreasing height of cell walls.

Numerical modeling by finite elements of a honeycomb structure is not easy but time consuming due to the geometrical complexity of the structure. Therefore, the homogenization for obtaining an equivalent homogeneous solid having the same global elastic properties is quite necessary in order to make numerical simulations more efficient.

Many studies have been performed on the analytical homogenization of honeycomb structures. The book of Gibson and Ashby [5] is the first systematic literature in this field. The in-plane elastic properties of honeycomb were first obtained with the beam theory. Further refinements by Masters and Evans [11] have been attempted considering stretching and hinging effects.

In the classical sandwich theory [12], the global skin–core interaction is identified as the result of the anti-plane core assumption. Since the two skins' constraints significantly alter the local deformation mechanism of the core, the homogenized core properties become sensitive to the ratio of the core thickness over the unit cell size. Due to the skins' constraint on the core's deformation, the core's stiffness varies non-linearly in function of the core thickness. This is called “core thickness effect” by Becker [13].

Chen et al. [14] have studied the thickness effect of a hexagonal honeycomb core on the elastic moduli by using the finite element method (FEM). The honeycomb bending deformation cannot be treated as a plate bending problem using the equivalent in-plane elastic modulus. Consequently, to calculate the honeycomb bending stiffness, Chen [15] has proposed a method based on the bending and twisting deformation of each wall of the open honeycomb. However, in his study, the skin effects were not studied. Becker [16] has given a closed form description with skin effect of the effective in-plane core stiffnesses including the thickness effect. His method was based on an approximate representation for the displacement field within the core cell walls, derived by considerations of energy.

Xu and Qiao [17] have proposed an analytical approach, called Multiple-Pass Homogenization (MPH), to homogenize a honeycomb unit cell including the skin and thickness effects. In their approach, the homogenization was carried out firstly along

X-direction (Fig. 1) to obtain two homogenous solids corresponding to inclined and vertical walls; then, a second homogenization was carried out along Y-direction to obtain the whole homogenous solid. This approach has been applied to four loading cases: transverse shear, in-plane stretch and shear, and bending. However, the interactions between the two homogenized solids were not equivalent to those between the vertical and inclined walls.

Following the study of Xu and Qiao [17], Qiao [18] has extended the application of the MPH technique to evaluate the equivalent torsion and in-plane shear stiffness properties of sandwich plates.

Chen et al. [19] have established another analytical model, with consideration of the skin and thickness effects, to calculate the moduli and the interfacial stresses for stretching problems. A solution using trigonometric function series based on the equilibrium equations has been proposed under the assumptions of some boundary conditions which limited their model to simple loading cases.

Xu [17] and Chen [19] have used hyperbolic functions (namely cosh and sinh) to describe the displacement field. It is worth mentioning that such functions may cause numerical problems if their argument increases (theoretically it can reach  $\infty$ ).

In our early research work [20], a homogenization formulation based on an energetic method has been proposed and successfully applied to study the influence of the sandwich skins and core's thickness on the core's elastic moduli in stretching and bending loading conditions. In this method, trigonometric functions series with unknown coefficients were used to describe the displacements field in the honeycomb core taking into account the skin and thickness effects. The minimization of the internal strain energy gave those coefficients and thus the elastic moduli. Furthermore, numerical models have been established to calculate the moduli of the homogenized equivalent solid, called “H-model”, for stretch, bending and stretch-bending coupled problems.

In this study, the above methodology is extended to study the in-plane shear, torsion, in-plane shear coupled with torsion, and transverse shear.

## 2. Methodology to determine the moduli of a homogenized honeycomb core

The homogenization consists in replacing the honeycomb structure by an equivalent homogenized solid. The energetic homogenization method is used in the present study to determine the elastic properties of the equivalent solid. The basic idea is that in a representative volume element (RVE, Fig. 1) of the honeycomb core, the internal strain energy  $\pi_{\text{int}}^*$  of the homogenized solid should be equal to that of the real cellular honeycomb core  $\pi_{\text{int}}$ :

$$\pi_{\text{int}}^* = \frac{1}{2} \int_{\bar{V}} \langle \varepsilon \rangle [E^*] \{ \varepsilon \} d\bar{V} = \pi_{\text{int}} = \frac{1}{2} \int_V \langle \varepsilon \rangle [E] \{ \varepsilon \} dV \quad (1)$$

where  $\bar{V}$  is the volume of the honeycomb RVE, and  $V$  denotes the total volume of all core's walls in the RVE. Eq. (1) is used to calculate the equivalent elastic moduli  $[E^*]$ . For the in-plane shear, torsion and transverse shear problems, the corresponding moduli, denoted  $G_{XY}^S$ ,  $G_{XY}^T$ ,  $G_{YZ}^S$  respectively, are all unique terms in the matrix  $[E^*]$ . Thus, they can be determined independently.

Suppose that the skins are submitted to a given displacement. Assuming a kinematically admissible displacement field of the core's walls with some coefficients to be determined, the corresponding strain field and internal strain energy  $\pi_{\text{int}}$  can be derived. The coefficients of displacement field are such that they minimize the internal strain energy. Finally, the latter can be evaluated, and the elastic moduli can be deduced using Eq. (1).

A honeycomb cell (Fig. 1) is taken as a RVE. Due to the triple symmetry of the structure, 1/8 RVE is considered. Fig. 2 shows its

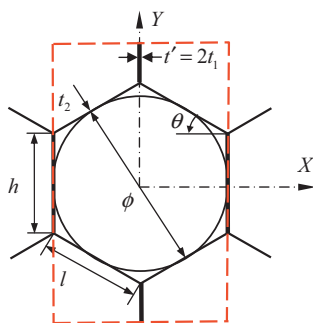


Fig. 1. Honeycomb RVE.

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