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Effect of drop-like aggregates on the viscous stress in magnetic suspensions

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ABSTRACT

We present results of theoretical and experimental study of effect of dense drop-like aggregates on the magnetoviscous effects in suspensions of non-Brownian magnetizable particles. Unlike the previous works on this subject, we do not restrict ourselves by the limiting case of highly elongated drops. This allows us to reproduce the experimental rheological curve in wide region of the shear rate of the suspension flow.

1. Introduction

Suspensions of magnetizable particles in carrier liquids, so-called magnetorheological suspensions (MRS), attract considerable interest of investigators and engineers due to rich set of physical properties which find active applications in many modern and perspective high technologies. One of the most interesting and valuable, from the practical point of view, features of MRS is possibility to change, in a very broad range, their rheological properties and behavior under the action of quite moderate magnetic fields. The physical cause of the strong rheological effects is formation of heterogeneous structures (aggregates) composed of magnetic particles and aligned with the applied field. Helpful reviews of the works on physics and practical applications of the magnetic suspensions can be found in [1–5].

In the quiescent suspensions, subjected to an external magnetic field, the aggregates can span the gap between the opposite walls of the flowing channel. In this state, MRS demonstrates elastic behavior with respect to the shear deformations. When the applied shear stress exceeds some threshold value (so-called static yield stress), the bonds between the aggregates and the walls are broken and the elastic regime changes to the viscous flow regime. In this regime, the measured macroscopical shear stress σ can be presented as

$$\sigma = \eta_0\dot{\gamma} + \sigma_a \tag{1}$$

Here $_0$ is the viscosity of the carrier liquid, $\dot{\gamma}$ is the macroscopic shear rate, σ_a is the stress produced by the aggregates. This stress is determined by the concentration, shape, length and orientation distribution of the aggregates.

Two kinds of the aggregates are usually considered. The first one is linear chains; the second kind of the aggregates is the dense bulk drops [1]. Theoretically, the effect of the chains on the stationary viscous properties of MRS with magnetizable particles has been studied in Refs. [6,7].

The effect of the bulk "drops", consisting of great number of particles, has been studied in Refs. [8,9]. The model [8] is based on minimization of the magnetic (electrical) free energy of a magnetizable (polarizable) ellipsoidal drop tilted with respect to the applied field. Analysis [8] has been done for highly elongated drops and leads to the following scaling relation: $\sigma_a \propto \dot{\gamma}^{1/3}H_0^{4/3}$, where H_0 is the magnetic field inside the suspension. It should be noted that the approach of Ref. [8] does not take into account any mechanisms of the drop destruction by the hydrodynamic viscous forces. Analysis shows that consideration of these mechanisms is principally important for development of a physically correct theory of rheological properties of suspensions with the heterogeneous aggregates.

The rupture of particles from the aggregated surface by the viscous forces has been considered in [9]. This model leads to the relation, $\sigma_a \propto H_0^2$; like in the chain model [6], the stress, σ_a, does not depend on the shear rate $\dot{\gamma}$. The model [9] is often used for the interpretation of rheological effects in the magnetic suspensions

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(see overview in Ref. [1]). However, this model contains a parameter (the thickness of a gap between the particles in the aggregates) which is not determined theoretically and is considered as a fit parameter of the model. Strictly speaking, the interparticle gap thickness must depend both on the field H_0 and the shear rate $\dot{\gamma}$, however in [9] this thickness is considered as a constant. Analysis shows that dependence of the gap thickness on the shear rate leads to qualitative change of the dependence of σ_a on $\dot{\gamma}$. Therefore, the results of [9] require very cautious attitude.

Like in Ref. [8], the model [9] deals with the limiting approximation of the highly elongated aggregates. However this approximation is not valid when the shear rate $\dot{\gamma}$ is large enough and too long aggregates are destroyed by the viscous forces.

We present here results of theoretical study of effect of the bulk drops on the magnetoviscous effects in MRS with the linearly magnetizable particles. The model is based on the analysis of hydrodynamic destruction forces acting on the aggregate surface and does not contain any adjustable parameters. It should be noted that appearance of the bulk drops has been observed in many experiments and computer simulations of magnetic suspensions (see for example, overview in [1]).

In the framework of this model we suppose that the drop size is significantly less than the width of the flow channel (the case of the developed flow) and effect of the channel walls on the drop behavior is negligible. The validity of this assumption will be proved at the end of the paper. Unlike Refs. [8,9] we do not restrict ourselves by the asymptote of the highly elongated drops. It allows us to reproduce the suspension rheograms in the wide range of the shear rates, $\dot{\gamma}$.

2. Main approximations

Let us consider a dense aggregate consisting of the large number of magnetizable particles. We will model the aggregate by an ellipsoid of revolution with the major and minor axes a and b, respectively. These magnitudes will be estimated below. We suppose that the suspension is subjected to a macroscopic shear flow with the rate $\dot{\gamma}$. When $\dot{\gamma} = 0$, the aggregate major axis is aligned along the magnetic field $\mathbf{H}_0$. Under the shear hydrodynamic forces, the ellipsoid axis deviates from the field by the angle θ. We will suppose that the field $\mathbf{H}_0$ is applied in the direction of gradient of the flow velocity. Next, we suppose that concentration of the drops in the suspension is small enough and any interactions between them can be ignored. The validity of this assumption will be reconsidered at the end of the paper. The problem geometry is illustrated in Fig. 1.

Macroscopic stress σ_a is determined by the shape of the drops and the angle θ of the drop inclination with respect to the field direction. On the other hand, the angle θ is determined by the hydrodynamic and magnetic torques acting on the aggregate; while the size and shape of the drop is defined by the combination of the hydrodynamic and magnetic forces acting on the drop surface. The magnetic force consists of the elongating force of the demagnetizing field along with the force of the surface tension. Obviously, the surface tension force tends to contract the drop. Our aim is to estimate the angle θ as well as the hydrodynamic and magnetic forces, and then – the size and the shape of the steady stable drop in the suspension under applied shear flow. For the maximal simplification of calculations we will consider this aggregate as a rigid ellipsoid, impenetrable for the carrier liquid, and restrict our analysis to a linear law of the particles magnetization.

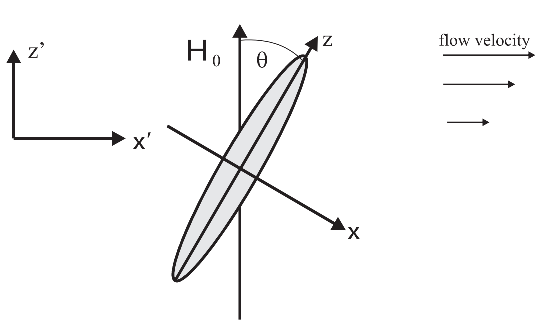

Fig. 1. Sketch of the model under consideration. Horizontal arrows illustrate the macroscopic velocity of suspension.

3. The orientation angle θ

The magnetic torque Γ_m, acting on the ellipsoidal drop, can be found, for example, in the book [10] (see also [8,9]) as:

$$\Gamma_m = \frac{1}{2} V \mu_0 \frac{\chi^2(1-3N)}{(1+N\chi)(2+(1-N)\chi)} H_0^2 \sin 2\theta \tag{2}$$

Here $V = \frac{4}{3}\pi ab^2$ is the aggregate volume, μ_0 is the magnetic permeability of vacuum, χ is the aggregate magnetic susceptibility, N is the aggregate demagnetizing factor in the direction of the main axis a. The explicit form of this factor can be found, in [10].

The hydrodynamic torque Γ_h has been calculated in Ref. [11] (see also [8]) as:

$$\Gamma_h = V\eta_0\dot{\gamma}\frac{4}{2r^2N+1-N}(r^2\cos^2\theta + \sin^2\theta) \tag{3}$$

Here $r = a/b$ is the drop aspect ratio, which will be determined below.

Equating the torques Γ_h and Γ_m, we get:

$$\tan\theta = \frac{BMa^{-1} - \sqrt{B^2Ma^{-2} - 4AC}}{2A}, \tag{4}$$

where $Ma = \frac{\eta_0\dot{\gamma}}{\mu_0 H_0^2}$ is the Mason number characterizing a ratio of the hydrodynamic-to-magnetic forces acting on aggregates,

$$A = \frac{4}{2r^2N+1-N}, \quad B = \frac{\chi^2(1-3N)}{(1+N\chi)(2+(1-N)\chi)}, \quad C = \frac{4r^2}{2r^2N+1-N}$$

4. Hydrodynamic destructing forces

We will estimate now a hydrodynamic force, which tends to elongate and, finally, to break the drop in the direction of the main axis a.

Let us introduce the Cartesian coordinate systems with the axes Oz' and Ox', aligned along the gradient of the suspension velocity (i.e. along the field $\mathbf{H}_0$) and along the velocity respectively; we also introduce another reference frame with the axes Oz along the major axis of the drop and the axis Ox situated in the plane $Ox'z'$. Both these coordinate systems are shown in Fig. 1.

In the laboratory ($Ox'z'$) frame the components of the flow velocity read:

$$v_{x'} = \dot{\gamma}z', \quad v_{y'} = v_{z'} = 0 \tag{5}$$

By using the standard formulas for transformations of vector components from one coordinate system to another, one can easily find the velocity components in the reference frame Oxz. After that we can determine the components of the rate-of-strain tensor in this frame:

$$\gamma_{xz} = \frac{1}{2}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) = \frac{1}{2}\dot{\gamma}(\cos^2\theta - \sin^2\theta), \tag{6}$$

$$\omega_{xz} = \frac{1}{2}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) = \frac{1}{2}\dot{\gamma} = -\omega_{zx}$$