Composite Structures 140 (2016) 525–533

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Wave propagation in hexagonal lattices with plateau borders

Kai Zhang [a], Zi-Chen Deng [a,*], Jun-Miao Meng [a], Xiao-Jian Xu [b]

[a] School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an 710072, PR China
[b] Key Laboratory for Special Area Highway Engineering of Ministry of Education, School of Highway, Chang'an University, Xi'an 710064, PR China

ARTICLE INFO

Article history:
Available online 8 January 2016

Keywords:
Hexagonal lattice
Plateau borders
Wave propagation
Dispersion relation
Band gap
Phase constant surface

ABSTRACT

The purpose of this paper is to investigate the plane wave propagation in two-dimensional hexagonal lattices with plateau borders and analyze the effects of the dimensional parameters on the dynamic properties of hexagonal lattices. Based on the Bloch's theorem and the finite element method, the dispersion relations of this structure are analyzed. The band gaps of three different structures are compared to illustrate the advantages of the hexagonal lattices with plateau borders. Effects of the dimensional parameters including the relative density, material distribution parameter and aspect ratio on the wave characteristics of the hexagonal lattices with plateau borders are also studied via the investigations of the gap maps and phase constant surfaces. More stop bands exist in the band structures of hexagonal lattices with plateau borders in comparing dispersion relations of regular hexagonal lattices with uniform thickness and lattices with parabolic thickness. The relative density, material distribution parameter and aspect ratio have significant effects on the wave characteristics of lattice structures. This work is expected to offer new opportunities for the multi-functional design of the hexagonal lattices and future applications in sound insulation devices.

1. Introduction

Two-dimensional prismatic cellular materials are attractive for the use in various engineering fields [1], such as light-weight structures [2,3], energy-absorbing guards [4], heat transfer devices [5,6], thermal protections [7], owing to their excellent specific stiffness and potential multi-functional applications [8]. A number of fundamental researches have been carried out on the mechanical behaviors [9,10] and optimizations of cellular structures in various working conditions [11,12]. Since the structural behaviors are strongly depend on their topologies, the attractive multi-functional applications of cellular structures are usually achieved by designing and optimizing the topologies and dimensions of the cellular materials. In recent years, the wave propagation properties of cellular materials and their applications as the sound insulation devices have been paid much attention as the periodicity of structures and designable unit cells [13,14].

In these 2-D structures, the periodically located impedance mismatch spaces, formed with the discontinuous geometries and/or materials of the structures, induce unique wave characteristics, in which propagations only exist over specific frequency bands, which is the so-called "stop-bands" or "band gaps". On the other hand, attenuation generated in other frequency bands is termed as "pass-bands". Therefore, the wave characteristics of periodic structures such as cellular structures [13], periodic 2-D composites [15] and chiral lattices [16] have attracted considerable attentions. Plane wave propagation in hexagonal and re-entrant lattices was investigated by Gonella and Ruzzene [17] to illustrate peculiar properties of re-entrant configurations and analyze the directional behaviors of hexagonal lattices with varying construction angles. A multi-functional structural design combining with superior mechanical wave filtering properties and energy harvesting capabilities was introduced by Gonella et al. [18] in which the dramatically enhancing filtering effect was achieved by introducing a microstructure consisting of stiff inclusions. Xu et al. [19] studied a special type of second-order hierarchical hexagonal lattice structures whose cell walls were replaced by some kind of structures and they found more band gaps in the low frequency ranges. Based on the above studies, it is known that the complex structural topologies might affect the mechanical wave characteristics and induce some interesting phenomena.

Generally, most theoretical models in describing the honeycombs assume that the thickness of cell walls is uniform and the material distribution is regular. However, it is known that the cellular materials with non-uniform thickness of cell edges [20] show their superior higher stiffness than those with uniform thickness for the same weight. The hollow-cylindrical-joint

* Corresponding author.
E-mail address: dweifan@nwpu.edu.cn (Z.-C. Deng).

http://dx.doi.org/10.1016/j.compstruct.2015.12.046
0263-8223/

honeycombs [21] and hierarchical honeycombs [22,23] are also the candidates in pursuing the outstanding mechanical performances in higher stiffness with constant masses. The above structures can also cause some locally resonant sonics areas which may induce the lower and wider band gaps. However, in view of the hexagonal honeycombs existing in the nature (see Fig. 1(a)), there are more materials locating near the junction of the cell walls. Meanwhile, as shown in Fig. 1(b), the pattern of idealized hexagonal honeycombs with plateau borders [24] looks much more close to this kind of natural honeycombs. Therefore, the structure closing to natural honeycombs may exhibit some fascinating mechanical properties [25] and is needed to provide ways of describing and analyzing its microstructure in the wave filtering properties. According to this point of view, we are concerned with investigating the wave characteristics of hexagonal lattices with plateau borders.

Motivated by the novel topology of such structure, we investigate the wave characteristics of hexagonal lattices with plateau borders, in which the effects of dimensional parameters on the band gaps and directional properties are analyzed.

The present study is organized in five sections including the Section 1 above. Section 2 describes the hexagonal lattices with plateau borders and their geometrical dimensions. Section 3 presents the approaches of the wave propagation analysis for the dispersion relations of the structures based on the finite element method (FEM) and Bloch's theorem. Section 4 analyzes the effects of the relative density, material distribution parameter and aspect ratio on the wave characteristics of hexagonal lattices with plateau borders. Section 5 finally summarizes the main results of this work.

2. Description of the hexagonal lattices with plateau borders

The sketch of a repeating unit cell of hexagonal lattices with plateau borders is shown in Fig. 2 as introduced in our previous work [25]. The cell angles between three cell edges in a junction are uniformly equal to 120° to imitate the typical natural honeycombs, and the three edges form a vertex with plateau borders which have a constant radius of curvature r_p.

The profile of each inclined cell edge is combined with two different types of cell edges with a total length $L/2$. Fig. 2 shows that Edge I is surrounded by the triangular box with dash lines and formed by the plateau borders with constant radius of curvature r_p and Edge II is the remaining part of a repeating unit cell. The cross-sectional thickness of Edge I can be expressed in terms of r_p. Meanwhile, the thickness of Edge II is assumed to be the quadratic function of x along the Edge II. Then, the thickness $t(x)$ with its local origin coordinate starting at point O of the two edges can be written as [25]

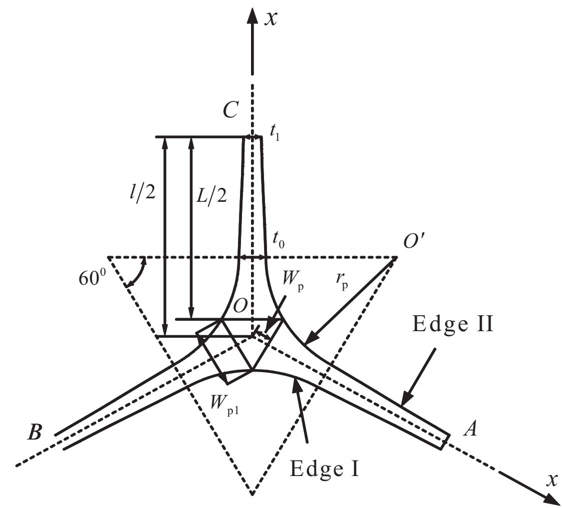

Fig. 2. A repeating unit cell and its dimension [25].

$$t(x) = \begin{cases} 2r_p + t_0 - 2\sqrt{r_p^2 - (x - r_p/2)^2} & 0 \leqslant x \leqslant r_p/2 \\ 2(x^2 + bx + c) & r_p/2 \leqslant x \leqslant L/2 \end{cases} \quad (1)$$

where $b = -\left(\frac{L+r_p}{2} + \frac{t_0 - t_1}{L - r_p}\right)$, $c = \frac{Lr_p}{4} + \frac{Lt_0 - r_p t_1}{2(L - r_p)}$. t_0 is the thickness of the cell at the beginning of Edge II and t_1 is the thickness of the cell at the end of Edge II. Additionally, we assume that the thickness of the cell edge decreases monotonically in Edge II ($r_p/2 \leqslant x \leqslant L/2$). In other words, $\frac{dt}{dx} = 4x + 2b < 0$. Because the vertex with plateau borders is considered in this work, the effective length of the cell walls (L) is different from the inclined cell length (l) and it can be obtained as $L = l - 2W_p$, where $W_p = \frac{1}{2}\left(\frac{2}{\sqrt{3}} - 1\right) r_p + \frac{1}{2\sqrt{3}} t_0$. Based on our previous work [25], the relative density is given by

$$\bar{\rho} = \frac{(2r_p + t_0)^2}{3l^2} - \frac{\sqrt{3}}{18l^2}\left\{\left(L^3 - r_p^3\right) + 3(r_p - L)\left[Lr_p - 2t_0(1+\beta)\right] + 4\pi r_p^2\right\} \quad (2)$$

with $\beta = t_1/t_0 \in [0, 1]$ being the material distribution parameter. Note that when $\beta = 1$, the cell wall thickness of honeycombs is uniform.

Based on the above analysis, we can also find that there are another two independent dimensional parameters in defining the geometry of hexagonal lattices, which are the radius of curvature of the plateau borders (r_p), the inclined cell length (l).

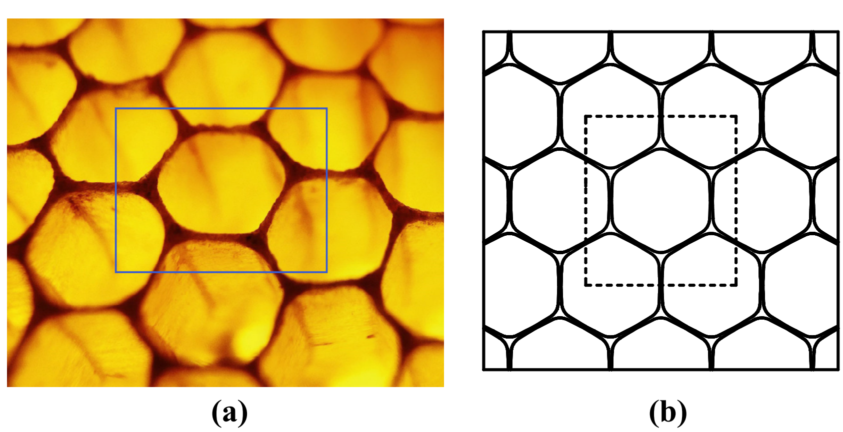

Fig. 1. (a) Hexagonal honeycombs existing in nature, (b) an idealized model for hexagonal honeycombs with plateau borders.