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The onset of purely elastic and thermo-elastic instabilities in Taylor–Couette flow: Influence of gap ratio and fluid thermal sensitivity

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ABSTRACT

Linear stability analysis of Taylor–Couette flow of dilute polymeric solutions has been performed by using two prototypical constitutive equations for polymeric solutions, namely, the Oldroyd-B and the FENE-P models. The hydrodynamic stability characteristics of the flow in the presence and absence of thermal effects and in the limit of vanishing fluid inertia have been determined using an eigenvalue analysis. Particular attention has been paid to accurately determine the instability onset conditions as a function of fluid thermal sensitivity and gap ratio. We observed a reduction in the critical Weissenberg, Wi_c, for the instability onset as the gap ratio and fluid thermal sensitivity are enhanced. In particular, under non-isothermal conditions, Wi_c was reduced by almost an order of magnitude for all gap ratios. Our results suggest that recent experiments leading to observations of "purely elastic turbulence" in the Taylor–Couette flow at order (1) *Wi* by Groisman and Steinberg (2004) [1] were not performed under isothermal conditions. Hence, this new flow state should be labeled "thermo-elastic turbulence."

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1. Introduction

Investigating the influence of elasticity on the hydrodynamic stability of prototypical flows has been a topic of much research in the past few decades [1–7]. In particular, curvilinear shear flows such as Taylor–Couette, Dean, cone-and-plate and plate-plate flows have received a great deal of attention as they are prone to purely elastic instabilities. In the absence of fluid inertia, purely elastic instabilities manifest as the Weissenberg number, *Wi*, which is defined as the ratio of elastic to viscous forces, becomes $O(1)$. Although spatio-temporal characteristics of instability and non-linear flow pattern transitions depend on flow geometry, purely elastic instabilities in curvilinear shear flows are attributed to significant polymer normal stress along the curved streamlines leading to "hoop" stresses. In turn, hoop stresses squeeze fluid elements radially inward, and elastic instability can be triggered [3,4,7].

The motion of a fluid confined between two infinitely long and concentric rotating cylinders is termed the Taylor–Couette flow for pioneering studies by Taylor [8]. These studies demonstrated a transition from a purely azimuthal flow to a secondary flow state above a critical Reynolds number, defined as the ratio of inertial to viscous forces. Almost seven decades later, Muller and coworkers observed a new mode of instability in the Taylor–Couette flow of dilute polymeric solutions at vanishing Reynolds number, *Re*, dubbed purely elastic instability [2]. In this pioneering study, the adverse gradient of elastic hoop stress across the curved streamlines was identified as the driving mechanism for the flow transition.

The aforementioned discovery was followed by several studies devoted to hi-fidelity prediction of the instability threshold condition [3,4,7,9]. Nonetheless, theoretical predictions and experimental observation [10–13] as well as experimental observations alone [3,10,11] have revealed qualitative and quantitative disparities. Specifically, time-dependent and asymmetric mode of instability predicted via isothermal linear and weakly non-linear stability analyses was observed to occur at critical *Wi* with an order of magnitude greater than experimentally measured values [3,12,13]. Moreover, it was indicated that secondary flow transition occurs via a subcritical bifurcation, i.e., there is a hysteresis in the flow pattern dynamic that cannot be predicted via linear stability analysis [13]. However, the predicted hysteresis loop could not adequately describe the discrepancies between linear stability

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results and experimental observations. A decade later, it was demonstrated that the coupling of viscous dissipation with thermal sensitivity of high viscous dilute polymeric solution gives rise to convection of the base-state temperature gradient by the radial perturbation velocity leading to a critical *Wi* an order of magnitude lower than that of the corresponding isothermal flow [14,15]. In fact, non-isothermal linear stability analysis predicted a symmetric and time-invariant instability mode, consistent with the instability flow patterns observed in experimental studies [10,11]. Furthermore, the non-linear stability analysis that Al-Mubaiyedh et al. implemented showed that the bifurcation corresponding to such thermo-elastic instability is supercritical, i.e., in a flow with the vanishing Reynolds number, the bifurcation leads to the axisymmetric and time-independent mode of instability [15].

While much effort was focused on resolving this aforementioned discrepancy, several experimental studies were performed on higher order non-linear flow transitions in the Taylor–Couette flow. Specifically, Groisman and Steinberg observed three dominant flow patterns in dilute PAAm aqueous solutions at high *Wi* and $O(1)$ elasticity number, $E = Wi/Re$, namely, diwhirls (DW), oscillatory strips (OS), and disordered states (DO) [16–18]. Subsequently, these flow patterns were reproduced via hi-fidelity direct numerical simulations (DNS), i.e., *Wi* and *E* corresponding to the transition from one flow state to another were faithfully reproduced [19,20]. In recent studies, elastically induced turbulent flow states have been also investigated via high fidelity DNS with the aim paving the way for understanding of this new class of flows [21,22].

Steinberg and Groisman were the first to observe a Taylor–Couette flow state at $O(1)$ *Wi* exhibiting broad range of spatial and temporal scales, named, "elastic turbulence" [1]. However, the influence of energetics on this novel flow state was not discussed, despite the fact that in their experiments, a dilute solution composed of a high molecular weight PAA $\left(18,000,000\left(\frac{gr}{grmol}\right)\right)$ in a highly viscous solvent of sugar, NaCl and water was utilized. Such high viscosity dilute polymeric solutions, commonly referred to as Boger fluids are prone to viscous dissipation when subjected to flow. Therefore, the fact that the "elastic turbulence" state, a higher order non-linear state, occurs at $Wi = 5$ and the lack of information on thermal effects in the experiments necessitates a more careful examination of instability onset conditions as a function of gap ratio and thermal sensitivity of the fluid.

This study examines the onset conditions for purely elastic and thermo-elastic instabilities in the Taylor–Couette flow over a wide gap ratio and fluid thermal sensitivity. This is accomplished via performing linear stability analysis with set of continuum conservation and constitutive equations. Specifically, the Oldroyd-B and the FENE-P constitutive models have been used to describe the rheological characteristics of dilute polymeric solutions. Our comprehensive analysis has clearly shown that although *Wi* for the onset of instability is reduced as the gap ratio is enhanced, without consideration of thermal effects, a $O(1)$ *Wi* instability condition cannot be realized. These findings cast doubt as to the existence of purely elastic turbulence in Taylor–Couette flows at $O(1)$ *Wi* as reported by Groisman and Steinberg [1].

2. Problem formulation

2.1. Governing equations

Fluid motion between two infinitely long and concentric cylinders of radii, R_1 and $R_2 (R_1 < R_2)$, is considered. The inner cylinder rotates with angular velocity of Ω_1, and the outer cylinder is stationary. The total solution viscosity, density, and the polymer solution's average relaxation time are noted as η_T, ρ and λ, respectively. The total solution viscosity is the sum of the solvent viscosity and polymeric contributions, i.e., $\eta_T = \eta_S + \eta_P$; where η_S and η_P refer to the solvent viscosity and polymeric viscosity, respectively.

In order to non-dimensionalize the governing equations, the gap width, $d = R_2 - R_1$, $d/R_1\Omega_1$, $R_1\Omega_1$, $\rho(R_1\Omega_1)^2$ and $\eta_P R_1 \Omega_1/d$ are chosen as the non-dimensionalizing scales for length, time, velocity, pressure, and polymeric stress, respectively. Additionally, the temperature is scaled with a reference temperature of T_0. In the flow system, the temperature difference is considered to be within the order of $O(1\,^\circ\mathrm{C})$ across the gap width, which is attributed to heat generation induced by viscous dissipation. Considering these conditions, the fluid is practically incompressible, leading to the following equation of continuity:

$$\nabla.\boldsymbol{v} = 0 \tag{1}$$

The non-dimensionalized equation of motion is expressed as

$$\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}.\nabla \boldsymbol{v}\right) = -\nabla P + \frac{1}{Re}\nabla.\{\beta\, e^{\epsilon_s\left(\frac{1}{T}-1\right)}(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^{\mathrm{T}}) + (1-\beta)\frac{1}{Wi}\boldsymbol{\tau}_p \tag{2}$$

In Eq. (2), P and $\boldsymbol{v}$ represent hydrodynamic pressure and flow velocity, respectively; $\boldsymbol{\tau}_p$ is the polymeric contribution to the stress. β is the ratio of solvent to total viscosity at the reference temperature, $\beta = \eta_s/\eta_T$; ϵ_s is dimensionless activation energy defined as

$$\epsilon_s = \frac{\Delta H_s}{R T_0^*} \tag{3}$$

In this equation, ΔH_s and R are activation energy attributed to the solvent and the universal gas constant, respectively. In Eq. (2), $\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T$ is defined as $\kappa = \nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T$, the rate of deformation tensor. In the same equation, the dimensionless Reynolds number appears as *Re*, which is defined as

$$Re = \frac{\rho R_1 \Omega_1 d}{\eta_T} \tag{4}$$

In the Oldroyd-B model, polymer molecules are modeled as non-interacting elastic dumbbells with a linear (Hookean) spring connector. The polymeric stress is formulated as the departure of the conformation tensor, $\boldsymbol{C}$, representing the ensemble average of the second moment of the polymer chain's end-to-end vector from its equilibrium conformation denoted as isotropic unit tensor I,

$$\boldsymbol{\tau}_p = \frac{\boldsymbol{C} - \boldsymbol{I}}{Wi} \tag{5}$$

in which $\boldsymbol{C}$ is non-dimensionalized with respect to a characteristic dumbbell length defined in terms of kT/H. Here k, T, and H correspond to the Boltzmann constant, temperature and Hookean spring constant, respectively.

Using the principle of time-temperature superposition and the concept of pseudo time [23], the isothermal Oldroyd-B constitutive equation for polymeric stress can be modified in a thermodynamically consistent fashion for the influence of thermal history on the stress and is given by

$$\begin{aligned}&\boldsymbol{\tau}_p + Wi.e^{\epsilon_\lambda\left(\frac{1}{T}-1\right)}\left[\boldsymbol{\tau}_{p(1)} - \left(\frac{\partial}{\partial t}(\ln T) + \boldsymbol{v}.\nabla \ln T\right)\boldsymbol{\tau}\right]\\ &\quad = 2e^{\epsilon_p\left(\frac{1}{T}-1\right)}\left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^{\mathrm{T}}\right)\end{aligned} \tag{6}$$

where $\boldsymbol{\tau}_{p(1)}$ is the upper convected derivative defined as

$$\boldsymbol{\tau}_{p(1)} = \frac{\partial \boldsymbol{\tau}_p}{\partial t} + \boldsymbol{v}.\nabla \boldsymbol{\tau}_p - (\nabla \boldsymbol{v})^{\mathrm{T}}\boldsymbol{\tau}_p - \boldsymbol{\tau}_p.\nabla \boldsymbol{v} \tag{7}$$

$\varepsilon_\lambda$and ε_p are the dimensionless activation energies for the relaxation time and polymer viscosity, respectively noted as

$$\epsilon_p = \frac{\Delta H_p}{R T_0^*} \; and \; \epsilon_\lambda = \frac{\Delta H_\lambda}{R T_0^*} \tag{8}$$

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