Composite Structures 139 (2016) 120–129

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Constant single-buckle imperfection principle to determine a lower bound for the buckling load of unstiffened composite cylinders under axial compression

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ARTICLE INFO

Article history:
Available online 15 December 2015

Keywords:
Buckling
Cylindrical composite shells
Single-perturbation load approach (SPLA)
Geometric imperfections
Metastability of cylindrical shells

ABSTRACT

Unstiffened cylindrical shells are very sensitive to geometric imperfections such as production-related deviations from the ideal geometry (conventional imperfections) as well as imperfect boundary conditions, material and wall thickness imperfections (non-conventional imperfections). The load carrying capability of unstiffened shells is reduced significantly by those imperfections. The NASA SP-8007 design guideline from 1968 is currently used for shell design. This guideline provides knock-down factors for the buckling loads and is based on experimental data of isotropic and orthotropic test specimen. Therefore the structural behavior of composite material is not considered adequately.

Based on the single-perturbation load approach (SPLA), this paper introduces a new physical based approach, to determine a lower bound for the buckling load of unstiffened composite cylindrical shells with respect to conventional and non-conventional imperfections, the constant single-buckle imperfection (CSBI) principle. The CSBI principle is based on the theory of the metastability of the geometric imperfect cylindrical shell which is introduced within this paper. The results indicate that the CSBI principle has the potential to provide an improved shell design in order to reduce weight and cost of thin-walled shells.

1. Introduction

First buckling experiments of thin, cylindrical shells were performed by Lundquist [1] in 1933 and Donell [2] in 1934. A significant number of buckling experiments were performed in the 1950s and 1960s. Weingarten, Morgan and Seide [3] summarized a series of experimental data, which can be seen in Fig. 1.

The experimental data is illustrated by means of a knock-down factor (KDF) ρ (ratio of experimental to theoretical buckling load) versus the slenderness of the shells (ratio of Radius R to wall thickness t). It can be seen that the theoretical determined buckling loads overestimate the experimental buckling loads in part by factor three to four. The difference between predicted linear buckling load and experimental buckling load provoked scientist to further investigate this problem in the last decades.

Koiter [5] showed in his dissertation from 1945 that geometric imperfections are responsible for the deviations between theoretical and experimental buckling load. Therefore subsequent investigations focused on geometric imperfections. Other different imperfection types like boundary condition imperfections (BCI), wall thickness and material imperfections (defined as stiffness imperfections – SI) have been gradually studied.

Geometric imperfections are defined as deviation from the ideal geometry of a cylinder and are further referenced as geometric mid-surface imperfections (MSI) after [6]. Geometric imperfections like welded and riveted joints are not considered within this paper.

1.1. Empirically derived design formulas

The buckling experiment results shown in Fig. 1 are the basis for the popular NASA SP-8007, a guideline for cylindrical shells, which was published in 1965 and revised in 1968 [7]. The knock-down factor ρ_{NASA} of the NASA SP-8007, which can be calculated using Eq. (1), is the result of a stochastic analysis and can be interpreted as a lower bound of the buckling load of the buckling experiments shown in Fig. 1. The corresponding design load N_{NASA} is determined by multiplying the KDF ρ_{NASA} with the buckling load N_{per} of a perfect shell (shell with ideal geometry, boundary conditions and material). This equation is only valid for isotropic and orthotropic materials. In Eq. (1) is the equivalent thickness t_{eq} that is

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http://dx.doi.org/10.1016/j.compstruct.2015.11.047
0263-8223/

Abbreviations and glossary

AI	axisymmetric imperfection
β	bending angle in [°]
β_{gmin}	critical bending angle of the CSBI principle in [°]
BCI	geometric boundary condition imperfection
BEAI	buckling eigenmode-affine imperfections
CSBI	constant single-buckle imperfection
F	axial load in [N]
FOSM	first-order second-moment method
KDF	knock-down factor
μ	probabilistic knock-down factor of the CSBI principle
MSI	geometric mid-surface imperfection
N	buckling load in [N]
N_{Exp}	experimental buckling load in [N]
N_{NASA}	design buckling load of the NASA SP-8007
N_{MSI}	buckling load of a cylinder with mid-surface imperfections in [N]
N_{per}	buckling load of a perfect cylinder in [N]
N_{SPLA}	buckling load of the SPLA (N_1) in [N]
N_{meta}	buckling load in the metastable equilibrium in [N]
N_{gmin}	buckling load in the global minimum in [N]
N_{CSBI}	design buckling load of the CSBI principle in [N]
P_1	perturbation load of the SPLA in [N]
ρ	knock-down factor in general
RF	reaction forces ($RF_{ERF} + RF_{GRF}$) in the bearing of the CSBI principle in [N]
RF_{ERF}	elastic restoring forces in [N]
RF_{GRF}	geometric response forces in [N]
SI	Stiffness Imperfections
SPLA	single perturbation load approach
v	indentation displacement in [mm]
v_{meta}	critical indentation displacement of the CSBI principle in [mm]
w	axial displacement in [mm]

commonly used to determine the KDF of an orthotropic material. The terms A_{11}, A_{22}, D_{11} and D_{22} are the extensional and bending stiffness's from the composite ABD matrix.

$$\rho_{NASA} = 1 - 0.902 \cdot (1 - e^{-x})$$

$$x = \frac{1}{16}\sqrt{\frac{R}{t}} \quad \text{(isotropic)} \tag{1}$$

$$x = \frac{1}{16}\sqrt{\frac{R}{t_{eq}}} \quad \text{with} \quad t_{eq} = 3.4689 \cdot \sqrt[4]{\frac{D_{11} \cdot D_{22}}{A_{11} \cdot A_{22}}} \quad \text{(orthotropic)}$$

The resulting KDFs of Eq. (1) cannot be used properly for composite shells because most of the stiffness terms of the composite ABD matrix are not considered, those terms have a significant influence on the buckling load which was shown by Geier et al. in [8]. Hühne et al. [9] show that the influence of imperfections on the buckling load depends on the ply-layup. In order to determine a state of the art KDF for composite cylinders the sensitivity to imperfections has to be considered.

1.2. Numerical design approaches for conventional imperfections

Substitute geometric imperfections can be used, in order to estimate the influence of geometric imperfections on the buckling load. A classification for those theoretical geometric imperfections as "realistic", "worst" and "stimulating" was proposed by Winterstetter and Schmidt [10] in 2002.

Arbocz [11] used measured geometric mid-surface imperfections, which consist of a cloud of measured points representing the mid-surface of the cylindrical shells. They deliver a very accurate representation of geometric mid-surface imperfections but for those "realistic" geometric imperfections the cylinder has to be manufactured and measured.

Buckling eigenmode-affine imperfections (BEAI) are "stimulating" and "worst" geometric imperfections and can for example be obtained by using a general purpose finite element code. In a subsequent non-linear simulation the buckling eigenmodes are applied in the cylindrical shell and the corresponding buckling load can be determined. A similar imperfection type is the axisymmetric imperfection (AI) which has been extensively investigated by Tennyson and Muggeridge [12] in 1969. Unfortunately there is no simple and unequivocal definition for both AI and BEAI regarding amplitude and number characteristic imperfections.

In 2005 Hühne [13] proposed a physically based deterministic concept for unstiffened cylindrical composite shells, the single-perturbation load approach (SPLA) which is illustrated in Fig. 2. The SPLA induces a single-buckle imperfection in the cylinder using a single-perturbation load, after a certain threshold of the perturbation load (P_1) the buckling load is almost constant, the corresponding buckling load is defined as N_1. A detailed investigation

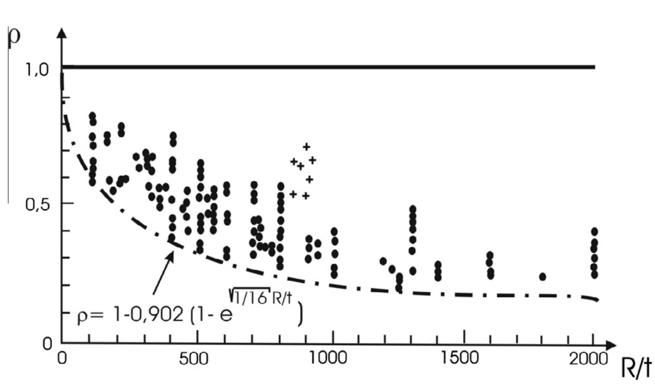

Fig. 1. Distribution of the experimental data of axial compressed cylindrical shells for different R/t ratios [4].

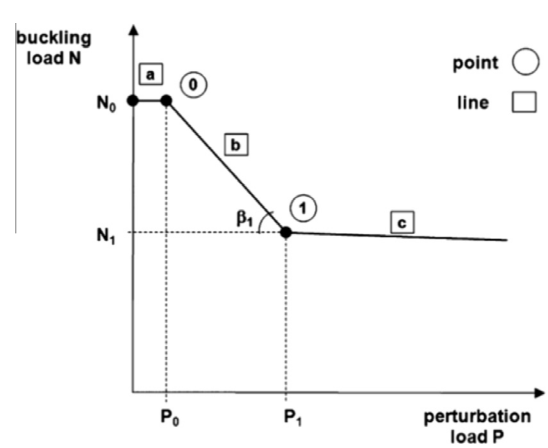

Fig. 2. Buckling load vs. perturbation load [11].