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The response of layered anisotropic tubes to centrifugal loading

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ABSTRACT

The displacement-based elastic solution for layered anisotropic tubes is extended to allow for the presence of centrifugal loading. The additional terms in the stress–strain equations derived in this work are validated by comparing the results obtained using the current solution against those determined using finite element simulation of rotating thin and thick-walled glass fibre reinforced plastic tubes of arbitrary anisotropic lay-up. The solution is presented in such a form that it can be utilised to determine the linear thermo-mechanical behaviour of rotating tubes with anisotropic lay-up, subjected to any combination of internal and external axisymmetric pressure, axial loading, torsional loading, and constant temperature change.

1. Introduction

In recent years fibre reinforced plastic (FRP) materials have been increasingly utilised because of the excellent strength-to-weight ratio and good corrosion resistance properties that they exhibit. One of the most important structures manufactured from these materials are tubes, which are widely used across many industries and in many different applications. FRP tubes are most widely used in the form of pipes for the transportation of corrosive substances, but they are also becoming frequently used as drive shafts in the automotive and aerospace industries. Depending on the stacking sequence of the plies of which a tube is comprised, the material properties may be transversely isotropic, orthotropic, or even anisotropic. The complex behaviour of such tubes has led to an appreciable body of research which is aimed at describing the elastic behaviour of these structures. Fortunately, due to the cylindrical nature of a tube, its elastic response is well defined, and numerous exact analytical solutions have been developed for FRP tubes under various loading conditions. One of the pioneering researchers in this regard was Lekhnitskii [1], and his work, which is based upon a stress function approach, has formed the basis of many analytical models [2–4] describing the response of composite tubes to different loads, and also for the measurement of residual stresses in cylindrically orthotropic tubes [5–8]. Stroh formalism [9] has also formed a fundamental platform upon which numerous analytical solutions [10–14] for composite tubes have been developed. Significant in these works is the state-space approach first presented by Tarn and Wang [13]. Another of the commonly utilised elastic solutions for the analysis of multi-layered, anisotropic tubes can be referred to as the displacement approach, and is fundamentally based upon, among others, the works of Sherrer [15], Reissner and Tsai [16], Wilson and Orgill [17,18], Kollár et al. [19], and Kollár and Springer [20]. This solution considers only axisymmetric thermomechanical loading, with the absence of out-of-plane shear forces. Under these circumstances, a generalised plane-strain condition arises, where the axial strain is constant over the wall thickness. This condition was initially realised by Leknitskii [1], and resonates in many subsequent works. The displacement-based elastic solution has been presented in many forms [21–25] and appears to have been first derived by Rousseau et al. [21]. The textbook of Herakovich [23] provides an in-depth derivation of this approach, which has also been extended to take account of material and geometric non-linearities [22], as well as through-thickness thermal variations [25].

None of the works mentioned thus far consider body forces associated with inertial loading. A common source of inertial loads in tubular structures is rotation about the longitudinal axis. Examples of composite tubes subjected, but not limited, to this type of loading are FRP drive shafts, flywheels, and centrifuge rotors. The practical importance of such structures means that there has, of course, been much research [26–35] focussed upon analytically describing the behaviour of rotating non-isotropic tubes. These works are, however, somewhat limited in terms of their application to layered FRP tubes. Firstly, many of these works [26–31] assume a state of plane-strain or plane-stress, and are therefore

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limited only to cases where these assumptions hold true. For instance, a plane-stress condition may arise should the component under consideration take the form of an annular plate or disc. On the other hand, a plane-strain condition may be assumed if the axial stiffness of a fairly long tube is much larger than the circumferential and radial stiffnesses. In general, however, these conditions seldom exist, and for an axisymmetric loading condition, a state of generalised plane-strain [23] arises. Secondly, most of these works [26–30,32–34] extend only to cylindrically orthotropic materials, and therefore cannot be utilised to analyse the elastic response of a rotating FRP tube of anisotropic lay-up. The only analytical solutions which address these deficiencies are the state-space approaches of Tarn and Wang [13] and Tarn [35]. Tarn and Wang [13] pointed out that centripetal acceleration gives rise to an axisymmetric state, and can be accounted for by adaptation of the governing equations that they presented. Tarn [35] later presented the state-space solution for rotating anisotropic functionally-graded tubes.

Although the state-space approach can be used to address the problem of centrifugal loads, it requires extensive matrix generation and eigen analysis. The displacement-based elastic solution offers a potentially less complex approach. It merely requires a number of constants, dependant on the number of layers in the tube, to be determined. These are easily found by considering the loading and the inter-laminar boundary conditions. It appears, however, that the displacement-based elastic solution has not yet been extended to account for the response of layered anisotropic tubes subjected to centripetal acceleration. The purpose of the present work is, therefore, to address this issue, and thereby provide an alternative means by which the behaviour of a rotating tube of this nature can be evaluated.

2. Theory

The derivation of this displacement-based elastic solution for a layered, anisotropic tube, subjected to centrifugal loading, is based upon the derivation presented in the textbook of Herakovich [23]. Consequently, the equations defining the through-thickness stress and strain distributions of the tube are identical to those presented by Herakovich [23], apart from additional terms which are associated with centripetal acceleration and which arise from the rotation of the tube about its central axis. The present derivation includes the terms associated with axisymmetric axial force, torque, internal and external pressures, as well as thermal effects arising from constant temperature change. Although the primary focus of the current work is on the effects of centrifugal loading only, the complete thermo-mechanical solution to the problem is provided so that the response of any rotating layered anisotropic tube, subjected to any combination of axisymmetric thermo-mechanical loading can be determined.

The laminated composite tube under consideration, comprising N layers, is illustrated in Fig. 1. The tube is assumed to be infinitely long, axisymmetic, and uniformly loaded along its length. The loads under consideration are internal and external pressures, P_I and P_O respectively, as well as axial and torque loads, F_x and T_x, which are associated with the integrals over the wall thickness of the axial stress, and the moment of in-plane shear stress, respectively [23]. The tube is assumed to be rotating about the central axis, x, with a constant angular velocity, Ω. Under the axisymmetric condition, all displacements, strains, and stresses are independent of the circumferential position, θ. Additionally, the radial displacements, w, are also independent of the axial position, x. The general expressions of the axial, u, circumferential, v, and radial, w, displacements of an arbitrary layer k can therefore be described by Eqs. (1)–(3) [23].

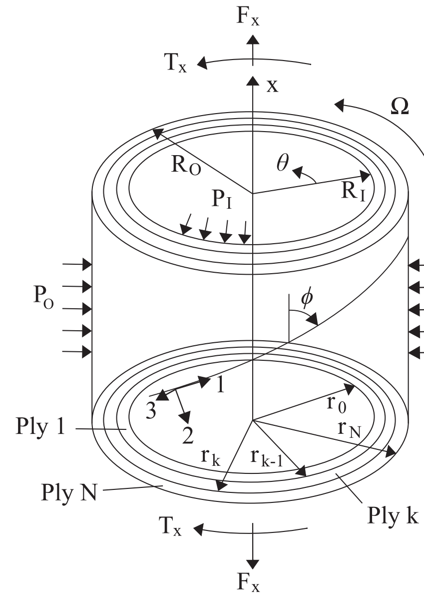

Fig. 1. Laminated composite tube.

$$u^k = u^k(x, r) \tag{1}$$

$$v^k = v^k(x, r) \tag{2}$$

$$w^k = w^k(r) \tag{3}$$

The strain–displacement equations for each layer k within an infinitely long tube uniformly loaded along its length are [23]:

$$\varepsilon_x^k = \frac{\partial u^k}{\partial x} \tag{4}$$

$$\varepsilon_\theta^k = \frac{w^k}{r} \tag{5}$$

$$\varepsilon_r^k = \frac{\partial w^k}{\partial r} \tag{6}$$

$$\gamma_{\theta r}^k = \frac{\partial v^k}{\partial r} - \frac{v^k}{r} \tag{7}$$

$$\gamma_{xr}^k = \frac{\partial u^k}{\partial r} \tag{8}$$

$$\gamma_{x\theta}^k = \frac{\partial v^k}{\partial x} \tag{9}$$

The compatibility equations, associated with axisymmetric displacement continuity are [23]:

$$\frac{d^2 \varepsilon_x^k}{dr^2} = 0 \tag{10}$$

$$\frac{1}{r}\frac{d\varepsilon_x^k}{dr} = 0 \tag{11}$$

$$\frac{1}{2}\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\gamma_{x\theta}^k\right)\right] = 0 \tag{12}$$

Integration of Eqs. (10) and (11) demonstrates that the axial strain within any layer k must be constant. Defining this constant as $\varepsilon_x^{0^k}$, the axial strain within this layer can be written as:

$$\varepsilon_x^k = \varepsilon_x^{0^k} \tag{13}$$

For an orthotropic layer k, the thermo-elastic constitutive equations in the principal material directions (1, 2, 3) are [23]: