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Vibration analysis of piezoelectric ceramic circular nanoplates considering surface and nonlocal effects

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ABSTRACT

Based on the theory of surface piezoelectricity and nonlocal piezoelectricity, a novel two-dimensional theory of piezoelectric nanoplates and boundary conditions are derived by utilizing the Hamilton's principle. The free and forced vibrations of piezoelectric ceramic circular nanoplates are first investigated with the derived two-dimensional equations. Closed-form solutions are obtained and surface effects are examined. The results presented in this paper can be reduced to some classical ones theoretically and numerically. It has been revealed that the surface and nonlocal effects have a great influence on the performance of piezoelectric circular nanoplates in terms of resonant frequency, displacement, stress, electrical potential, current, capacitance ratio, and other quantities. A critical thickness of piezoelectric plate is calculated for the first time, below which the size-dependent effect is obvious that must be considered. These findings can provide effective guidance for the explanation of certain physical phenomenon about size-dependent electromechanical characteristics and experimental design of piezoelectric devices in nanoscale.

1. Introduction

Piezoelectric materials have been used to make various smart devices due to their intrinsic electromechanical coupling effect, such as sensors, resonators, actuators, transducers, harvesters and so on. With the fast development of nanotechnology, piezoelectric nanostructures have attracted tremendous attention due to their superior physical properties and potential applications in modern science and technology [1,2]. For example, researchers [3,4] have demonstrated a novel concept of nanogenerator based on one-dimensional ZnO nanowires arrays, which can convert efficiently the nanoscale mechanical energy into the electric energy. In a nanomaterial body, the surface or near-surface atoms are usually subjected to different environmental constrains from their bulk counterparts. Therefore, the surface layer may play a distinguished role in determining the material behavior of the nanostructure when the aspect surface-to-volume ratio increases. In other words, nanomaterials exhibit obvious size-dependent phenomena, which have been demonstrated in recent experimental and theoretical studies [5,6].

Currently, three main approaches are utilized to study nanomechanics: experimental test, molecular dynamic simulation and the nonclassical continuum theory. The size-dependent characteristics of nanomaterials, though, can be well predicted from molecular dynamic simulation or experimental tests, by using the continuum theories, the analysis of nanostructures, will allow us to better understand their size-dependent effects. For example, by using modified continuum theories based on a well-known surface elasticity model developed by Gurtin and Murdoch [7], the size-dependent properties of elastic nanobeams [8,9] and nanoplates [10,11] have been well investigated. However, due to this fact that surface elasticity model neglects surface piezoelectricity effect, which is unique for piezoelectric materials. Hence, it is not sufficient in predicting the size-dependent properties of piezoelectric nanomaterials. To better reveal the surface effects of piezoelectric nanomaterials, Huang and Yu [12] firstly proposed a surface piezoelectricity model which is an extension of the surface elasticity theory. Following the surface piezoelectricity model developed by Huang and Yu and Euler–Bernoulli beam theory, the influences of surface effects on the electromechanical coupling and the bending behavior of nanowires as well as the vibration and buckling behaviors of piezoelectric nanobeams were discussed by Yan and Jiang [13,14]. Further, they investigated the static and dynamic

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behavior of piezoelectric nanoplates by considering surface effects and combining the Kirchhoff plate theory [15–17].

It is worth mentioning that the nonlocal elasticity theory was initiated by Eringen [18] and has been widely utilized to analyze bending, buckling, vibration, and wave propagation of nanostructure [19,20]. Recently, Zhou et al. [21] extended Eringen's nonlocal elasticity theory to piezoelectric materials. Based on the nonlocal piezoelectricity theory proposed by Zhou et al. and Timoshenko beam theory, the thermo-electro-mechanical linear and nonlinear vibrations of piezoelectric nanobeams were discussed by Ke et al. [22,23]. Later, they [24] investigated the thermo-electro-mechanical vibration of piezoelectric cylindrical nanoshells by using the nonlocal Love's thin shell theory. Liu et al. [25] studied the thermo-electro-mechanical free vibration of piezoelectric nanoplates based on the nonlocal Kirchhoff plate theory. Further, the nonlocal piezoelectricity theory was extended to magneto-electro-elastic (MEE) composite materials in order to investigate the size-dependent vibration and buckling characteristics of MEE nanostructures. For example, the free vibration of MEE nanobeams [26], nanoplates [27] and nanoshells [28] were investigated by Ke et al. based on the nonlocal Timoshenko beam theory, Kirchhoff plate theory and Love's shell theory, respectively. By considering the influences of both surface effect and nonlocal effect, Wang and Wang [29] analyzed the electromechanical coupling behavior of piezoelectric nanowires. Zhang et al. [30] studied the dispersion characteristics of elastic waves in a monolayer piezoelectric nanoplate based on surface and nonlocal effects.

The theoretical modeling, mechanical analysis and physical explanation of piezoelectric nanostructures, such as nanoscale piezoelectric beam, plate, shell, wire and tube, which are usually some basic components of nano-devices, are essential for their design and applications. Lately, some novel theories were proposed by many researchers which can be used to predict electromechanical coupling behaviors of nano-devices. For instance, Chen [31] developed a novel surface piezoelectricity theory in which the surface layer is assumed to be of small thickness by using the concept of effective boundary conditions for the bulk materials and employing the state-space formulations. Zhang et al. [32,33] established a two-dimensional theory of piezoelectric plates and shells by considering surface effects. Chen et al. [34] discovered that both surface and nonlocal effects have a remarkable influence on the behaviors of the nanomaterials. However, to date, the relevant literatures are very limited except the works of Wang and Wang [29] and Zhang et al. [30]. In addition, to the best of the authors' knowledge, no literature is available so far for the piezoelectric circular nanoplates by combining surface and nonlocal effects. Only few works [11] were reported about the vibration characteristics of circular nanoplates based on the surface elasticity theory. However, this is significant for the design of high quality nano-electronic devices. Consequently, it is necessary and significant to investigate the electromechanical coupling behaviors of piezoelectric circular nanoplates by considering the surface and nonlocal effects.

The objective of the paper is to establish a novel two-dimensional theory of piezoelectric plates by considering both surface effects and nonlocal effects to analyze the free and forced vibrations of piezoelectric ceramic circular nanoplates. The basic equations of nonlocal piezoelectricity and surface piezoelectricity are given in Section 2. In the following, two-dimensional equations of piezoelectric nanoplates and boundary conditions are derived in Section 3 by utilizing the Hamilton's principle as well as considering surface and nonlocal effects. The free and forced vibrations of piezoelectric nanoplates are discussed in detail in Sections 4 and 5, respectively. Finally, some conclusions are proposed in Section 6.

2. Theory of nonlocal piezoelectricity and surface piezoelectricity

In the theory of nonlocal piezoelectricity, the essence of this theory is that the stress tensor and the electric displacement at a reference point depend not only on the strain components and electric field components at the same position but also on all other points of the body. Mathematically, the basic equations for a homogeneous and nonlocal piezoelectric solid with zero body force can be written as [22,29]

$$T_{ij} = \int_V \alpha(|x' - x|, \tau) T'_{ij}(x') dV(x'), \quad D_i = \int_V \alpha(|x' - x|, \tau) D'_i(x') dV(x'), \tag{1}$$

$$T'_{ij}(x') = c_{ijkl} S_{kl}(x') - e_{kij} E_k(x'),$$
$$D'_i(x') = e_{ikl} S_{kl}(x') + \varepsilon_{ik} E_k(x'), \quad i, j, k, l = 1, 2, 3 \tag{2}$$

$$T_{ij,i} = \rho \ddot{u}_j, \quad D_{i,i} = 0, \tag{3}$$

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\phi_{,i}, \tag{4}$$

where T_{ij}, T'_{ij}, S_{ij}, D_i, D'_i, E_i, u_i and ϕ are the nonlocal stress, local stress, strain, nonlocal electric displacement, local electric displacement, electric field, displacement components and electric potential, respectively; c_{ijkl}, e_{kij}, ε_{ik} and ρ are the elastic constants, piezoelectric constants, dielectric constants and mass density, respectively. $\alpha(|x' - x|, \tau)$ is the nonlocal attenuation function which incorporates into the constitutive equations at the reference point x produced by the local strain at the source x' and the nonlocal modulus satisfying $\int_V \alpha(|x' - x|, \tau) dV(x') = 1$. $|x' - x|$ is the Euclidean distance. $\tau = e_0 \bar{a}/L$ is defined as the scale coefficient that incorporates the small scale factor, where e_0 is a material constant determined experimentally or approximated by matching the dispersion curves of the plane waves with those of the atomic lattice dynamics; and $\bar{a}$ and L are the internal (e.g. lattice parameter, granular size) and external characteristic lengths (e.g. crack length, wavelength) of the nanostructures, respectively.

According to Eringen [18], the nonlocal constitutive behavior in the spatial integral forms can be represented by the follow differential constitutive equations

$$(1 - \mu \nabla^2) T_{ij} = T'_{ij} = c_{ijkl} S_{kl}(x') - e_{kij} E_k(x'), \tag{5}$$

$$(1 - \mu \nabla^2) D_i = D'_i = e_{ikl} S_{kl}(x') + \varepsilon_{ik} E_k(x'), \tag{6}$$

where $\mu = (e_0 \bar{a})^2$ and ∇^2 are the nonlocal parameter and Laplace operator, respectively.

For a piezoelectric solid of volume V and bounded by surface S, Eq. (3) and the corresponding boundary conditions can be derived from the Hamilton's principle [35]

$$\delta \int_{t_0}^{t_1} dt \int_V (K - H) dV + \int_{t_0}^{t_1} dt \int_S (t_j \delta u_j + \sigma \delta \phi) dS = 0, \tag{7}$$

where $K = \frac{1}{2} \rho \dot{u}_j \dot{u}_j$ is the kinetic energy density, $H = \frac{1}{2} T_{ij} S_{ij} - \frac{1}{2} D_i E_i$ is the electric enthalpy density, t_j and σ are the surface traction and the surface charge density, respectively.

The corresponding constitutive Eq. (1) are related to the electric enthalpy density by

$$T_{ij} = \frac{\partial H}{\partial S_{ij}}, \quad D_i = -\frac{\partial H}{\partial E_i}. \tag{8}$$