Composite Structures 137 (2016) 44–55

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Buckling of laminated-glass beams using the effective-thickness concept

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ARTICLE INFO

Article history:
Available online 11 November 2015

Keywords:
Laminated glass
Structural composites
PVB
Buckling
Structural stability
Viscoelasticity

ABSTRACT

Structural stability is one of the design requirements in laminated-glass beams and plates due their slenderness and brittleness. In this paper the equations of the classical Euler theory for buckling of isotropic monolithic beams are extended to laminated-glass beams using the effective thickness and the effective Young modulus concepts. It is demonstrated that the dependency of the effective stiffness on boundary conditions can be considered using buckling ratios of Euler theory corresponding to isotropic linear monolithic beams. The analytical predictions are validated by compressive experimental tests in simply supported beams. Fixed boundary conditions are difficult to reproduce in experimental tests due to the brittleness of the glass and for this reason fixed–fixed and fixed–pinned boundary conditions were validated using a finite element model.

1. Introduction

Laminated glass is a sandwich or layered material which consists of two or more plies of monolithic glass with one or more interlayers of a polymeric material with mechanical properties that are time (or frequency) and temperature dependent [1]. Polyvinyl butyral (PVB) is the most widely used interlayer material, although the new ionoplastic interlayers improve the mechanical properties of laminated glass for a broad range of temperatures [1]. Polyvinyl butyral (PVB) is sold in thicknesses of 0.38 mm or a multiple of this value (0.76 mm, 1.12 mm, and 1.52 mm).

Laminated glass is easy to assemble in a finite element model but many small 3D elements are needed to mesh accurately because the thickness of the viscoelastic interlayer is usually very small compared with the dimension of the laminated-glass element. Cubic elements in 3D and square elements in 2D generally result in equations that are well conditioned but if the element shape is greatly distorted from these ideal shapes, numerical difficulties can arise [2]. If we wish to mesh the interlayer of a square laminated-glass plate 2000 mm $\times$ 2000 mm with 2 cubic elements along the thickness, we would need approximately 27.7×10^6 elements only to mesh the interlayer. Moreover, if a quasi-static analysis is performed taking into account the temperature and time-dependent behavior of the interlayer, the time needed to perform the calculation is considerably higher than that needed for a static analysis. Consequently, the 3D models in laminated-glass elements are very costly in time and memory.

The calculation of laminated-glass elements can be facilitated by simplifying the viscoelastic solution using the quasi-elastic method, which consists of describing the viscoelastic behavior of the interlayer by an elastic behavior with parameters that depend on the load duration and temperature [3–7]. This means that the memory effect of the viscoelastic material is neglected and that the mechanical properties are linear elastic but time dependent [7–9].

The concept of effective thickness has been proposed in recent years [7,9,10] based on the quasi-elastic solution. This method consists of calculating the thickness (time and temperature dependent) of a monolithic element with bending properties equivalent to those of the laminated one, that is, the deflections provided by the equivalent monolithic beam are equal to those of the layered model with a viscoelastic core. The effective thickness can then be used in analytical equations and simplified finite element models in place of the layered laminated-glass element [7,9–11]. The effective-thickness concept is proposed in most of the technical standards related to laminated glass and it is more readily applicable in design practice. The effective-thickness concept is not easy to implement in finite element programs because a monolithic model with constant Young modulus and a temperature- and time-dependent thickness has to be defined. As the effective thickness is derived from the effective stiffness [7,9,10], an effective Young modulus [11] can also be inferred from the effective stiffness, this being more attractive to be used in numerical models (a monolithic model with constant thickness is defined whereas the Young modulus is time and temperature dependent). Thus,

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http://dx.doi.org/10.1016/j.compstruct.2015.11.014
0263-8223/

Nomenclature

E_{eff}	effective Young modulus
E	Young modulus of glass layers
$E_2(t)$	viscoelastic relaxation tensile modulus for polymeric interlayer
$G_2(t)$	viscoelastic relaxation shear modulus for the polymeric interlayer
H_1	thickness of glass layer 1 in laminated glass
H_2	thickness of polymeric layer 2 in laminated glass
H_3	thickness of glass layer 3 in laminated glass
H_{TOT}	$H_1 + H_2 + H_3$
H_0	$H_2 + \left(\frac{H_1+H_3}{2}\right)$
I	second moment of area
I_1	$\frac{H_1^3}{12}$
I_3	$\frac{H_3^3}{12}$
I_T	$I_1 + I_3 = \frac{H_1^3+H_3^3}{12}$
I_{TOT}	$I_T(1+Y)$
$K_2(t,T)$	viscoelastic bulk modulus
L	length of a glass beam
$P_{crit}(t,T)$	critical load
T	temperature
T_0	reference temperature
Y	$\frac{H_0^2 H_1 H_3}{I_T(H_1+H_3)}$

Lowercase letters

a_T	shift factor
b	width of a glass beam
$g(x)$	shape function (Galuppi and Royer Carfagni model)
t	time
w	deflection

Greek letters

η_2	loss factor of the polymeric interlayer of laminated glass
ν	Poisson ratio of the glass layers
$\nu_2(t,T)$	viscoelastic Poisson ratio of the polymeric interlayer

the effective-thickness and the effective Young modulus concepts can be used interchangeably with the same accuracy.

The effective-thickness concept allows also stress-effective thickness to be defined, i.e. the thickness of a monolithic beam with equivalent bending properties in terms of stresses. However, due to the fact that the buckling behavior is governed by its flexural stiffness, only the deflection effective thickness is considered in this paper.

If laminated-glass elements are subject to compressive loads, structural stability is one of the design requirements because laminated-glass elements are brittle and slender. Due to the fact that the stiffness of the interlayer is temperature and time dependent, the same is true of the critical load, that is, the critical load of a laminated-glass beam subject to constant compressive load decreases with time.

Several analytical models have been proposed for determining the critical load of a simply supported laminated-glass beam [12–15] but only a few are devoted to other boundary conditions [16]. In monolithic beams, the effect of the boundary conditions is considered through the buckling ratio β (or alternatively with the effective length L_{eff}) whereas the stiffness EI is constant. In this paper, we demonstrate that the effective stiffness also depends on the boundary conditions and its effect can also be taken into account through the buckling ratio β.

The aim of this paper is to propose a simplified method to calculate critical loads in laminated-glass beams with different boundary conditions using the Euler theory [17] of monolithic beams, the quasi-elastic solution [8,9] and the effective-stiffness concept [7–10]. As a means of validating the model, the critical load of several laminated-glass beams, made of annealed glass plies and a PVB core, were predicted using the effective stiffness concept and validated by experimental tests and numerical models.

1.1. The effective-thickness concept

The concept of effective thickness for calculating deflections in laminated-glass beams under static loads was proposed by Calderone et al. [7] based on a previous work of Wölfel [18]. Later, Galuppi and Royer-Carfagni [9] derived new equations for the deflection effective thickness using a variational approach and assuming that the deflection shape of the laminated-glass beam coincides with that of a monolithic beam under the same load and boundary conditions; that is, the deflection of the beam is assumed to be:

$$w(x,t,T) = -\frac{g(x)}{EI(t,T)_S} \tag{1}$$

where $g(x)$ is a shape function that takes the form of the elastic deflection of a monolithic beam with constant cross section under the same load and boundary conditions as the laminated-glass beam and where $EI(t)_S$ is the bending stiffness of the laminated-glass beam given by:

$$EI(t,T)_S = \frac{1}{\frac{\eta_S(t,T)}{EI_T(1+Y)} + \frac{1-\eta_S(t,T)}{EI_T}} \tag{2}$$

where:

$$\eta_S(t,T) = \frac{1}{1 + \frac{EH_1H_2H_3\psi_B}{(1+Y)G_2(t,T)(H_1+H_3)}} \tag{3}$$

The parameter ψ_B [9] can be expressed as:

$$\psi_B = \frac{\gamma}{L^2} \tag{4}$$

with γ being a constant parameter which depends on the boundary and load conditions [9].

Calderone et al. [7] proposed an effective stiffness for a laminated-glass beam subjected to static loads, which is expressed as:

$$EI(t,T)_S = EI_T(1 + \Gamma_S(t)Y) \tag{5}$$

where

$$\Gamma_S(t,T) = \frac{1}{1 + 9.6\frac{EH_1H_2H_3}{G_2(t,T)(H_1+H_3)L^2}} \tag{6}$$

Eqs. (2) and (5) can be expressed in a unified form as

$$EI(t,T)_S = EI_T\left(1 + \frac{Y}{1 + \gamma\frac{EH_1H_2H_3}{G_2(t,T)(H_1+H_3)L^2}}\right) \tag{7}$$

Eq. (6) proposed by Calderone et al. [7] is based on a previous work of Wölfel devoted to composite sandwich structures under various boundary and loading conditions, leading to different