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Free vibration of anti-symmetric angle-ply plates with variable thickness

Saira Javed [a,b], K.K. Viswanathan [a,b,*], Z.A. Aziz [a,b], Kandasamy Prabakar [c]

[a] UTM Center of Industrial and Applied Mathemathics (UTM-CIAM), Ibnu Sina Institute for Scientific and Industrial Research, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

[b] Department of Mathematical Science, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

[c] School of Electrical Engineering, Pusan National University, San 30, Jangjeong-Dong, Gumjeong-Ku, Busan 609 735, South Korea

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ABSTRACT

Free vibration of anti-symmetric angle-ply plates with variable thickness is analysed using spline function approximation including the effect of shear deformation. The equations of motion for the plate are derived using the theory of Yang, Norris and Stavsky. Assuming the solution in a separable form, a system of coupled differential equations in displacement and rotational functions are obtained and these functions are approximated by Bickley-type splines of order three. A generalised eigenvalue problem is obtained and solved numerically for an eigenfrequency parameter and an associated eigenvector of spline coefficients. Two and four layered plates consisting of two different materials and plies comprising of same as well as different materials for two different boundary conditions are analysed. The effect of material properties, ply orientation, number of lay ups, aspect ratio and coefficients of thickness variations on the frequency parameter are presented. The accuracy of the result is ascertained by convergence and comparative study.

1. Introduction

Plates with variable thickness are usually used in numerous engineering applications. The significant characteristics of plates of variable thickness is to alter the frequency and to reduce the weight and size of the structure. Moreover, plates with non-uniform thickness provides greater efficiency for vibration than plates of constant thickness.

Aeronautical, aerospace, marine engineering and various modern technologies are increasingly using composite laminated structures. The strength and deformation of such structural elements are influenced by ply orientation, stacking sequence and lamination material. The main aim of the designer is to control unwarranted (undesirable) vibration which in return leads to the failure of the structure. Composite materials have the ability to tailor the mechanical properties. Further they offer high stiffness to weight ratio, strength to weight ratios, better temperature resistant and shock absorbing characteristics.

Laminated composite plates are usually analysed using classical plate theory (CPT) and first order shear deformation theory (FSDT). The transverse shear deformation is omitted in CPT, so it can only analyse thin plates accurately. Whereas, composite plates have low transverse shear modulus relative to the in-plane Young's moduli due to which transverse shear effects are more noticeable. FSDT is commonly used to analyse the composite laminated plates. The Yang et al. [1] was first to develop a shear deformable theory for laminated anisotropic plates which is actually generalisation of Mindlin's theory.

Several articles on laminated composite plates can be seen in the literature by various researchers, among them are Leissa [2], Reissner [3], Bert and Chen [4], Reddy [5], Ferreira and Fernandes [6], Qatu [7], Soedal [8], Szilard [9] and Chakraverty [10]. Various methods are available for the analysis of plates such as Navier method was used by Aghababaei and Reddy [11], Mantari et al. [12], Thai and Choi [13] and Messina and Soldatos [14]. Moreover, Thai and Kim [15], Bai and Chen [16] and Hashemi et al. [17] analysed the free vibration of plates by applying Levy's method. Elmalich and Rabinovitch [18], Kucukrendeci and Kucuk [19] and Thai et al. [20] adopted finite element method for analysing the free vibration of plates. Narita [21], Fazzolari and Carrera [22] and Fiorenzo et al. [23] carried out vibration analyses of plates based on Rayleigh–Ritz method. Ding et al. [24] and Chen and Lue [25] presented state-space method for analysing the free vibration of laminated rectangular plates. Shu [26] discussed differential quadrature method and its engineering applications. Further, Liew et al. [27], Ferreira et al. [28] and Kamarian et al. [29] studied the

* Corresponding author at: Department of Mathematical Science, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia.

E-mail address: visu20@yahoo.com (K.K. Viswanathan).

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vibration of laminated plates based on differential quadrature method. Moreover, free vibration of laminated plates problems were solved using Meshless method by Ferreira et al. [30] and Xiang and Kang [31]. Roque et al. [32], Ferreira [33] and Ferreira and Fasshauer [34] computed the frequencies of plates by using radial basis functions. Fazzolari and Carrera [35] and Liu et al. [36] used Galerkin method for analysing vibration of plates. Discrete singular convolution method was adopted by Zhu and Wang [37] and Wang et al. [38] for solving the plate's vibrational problem. Vibration of plates was analysed using extended Kantorovich method by Rahbar and Rostami [39] and Fallah et al. [40]. Mixed variational formulations were adopted to analyse the free vibration of plates by Yu [41], Kim [42] and Phoenix et al. [43]. Moreover, Noor [44], Demasi [45], Liu and Xing [46] and Messina [47] used exact solution for solving the problems of free vibration of plates. In this context, we used spline method for the free vibration analysis of antisymmetric angle-ply plates of variable thickness.

In addition to that antisymmetric angle ply laminates were studied by Patil [48] under various shear deformation theories. Further, superposition method was used to analyse the free vibration of unsymmetric cross-ply and antisymmetric angle-ply functionally graded plates by Kshirsagar and Bhaskar [49]. Simply supported cross-ply and antisymmetric angle ply plates were analysed using first-order shear deformation theory by Thai and Choi [13] and Sadoune et al. [50]. Free vibration of anti-symmetric laminated thin square plate was analysed by Aydogdu and Timarci [51] using Ritz method.

Some of the researchers analysed the plates having non-uniform thickness such as linear thickness variation of plates was considered by Bambill et al. [52] and Civalek [53], parabolic thickness variation by Xu et al. [54], quadratic thickness variation by Grigorenko et al. [55], cubic thickness variation by Zenkour [56] and exponential thickness variation by Lal [57].

The present wok analyse the free vibration of antisymmetric angle-ply plates including shear deformation theory. The thickness variation is considered in x direction which is assumed to be linear, exponential and sinusoidal. A system of coupled differential equations consisting of three displacement and two rotational functions are approximated by cubic splines. A spline technique is used due to the possibility, that a chain of lower order approximations which can yield a greater accuracy than a global high order approximation [58]. Collocation with these splines yields a set of field equations which, along with the equations of boundary conditions, reduce to a system of homogeneous simultaneous algebraic equations on the assumed spline coefficients. The resulting generalised eigenvalue problem is solved for a frequency parameter, using eigensolution technique, to obtain as many frequencies as required, starting from the least. From the eigenvectors, the spline coefficients are computed from which the mode shapes can be constructed. Parametric studies includes the effect of aspect ratio, ply angle and three different thickness variations on the frequency parameter of two and four layered plates consisting of two different materials. Numerical results are presented in terms of graphs and tables.

2. Formulation of the problem

Consider a rectangular plate in the Cartesian coordinate system x, y and z with the xy plane placed at mid-depth (reference surface) of the plate and z is taken to be normal to the plate shown in Fig. 1. Here a and b are the lengths of the sides of the plate along x and y directions respectively, h is the thickness of the plate and h_k is the thickness of the kth layer.

The equations of motion in terms of stress and moment resultants (neglecting body forces) are

$$N_{x,x} + N_{xy,x} = P\frac{\partial^2 u}{\partial t^2}$$

$$N_{xy,x} + N_{y,y} = P\frac{\partial^2 v}{\partial t^2}$$

$$Q_{x,x} + Q_{y,y} = P\frac{\partial^2 w}{\partial t^2}$$

$$M_{x,x} + M_{xy,y} - Q_x = I\frac{\partial^2 \psi_x}{\partial t^2}$$

$$M_{xy,x} + M_{y,y} - Q_y = I\frac{\partial^2 \psi_y}{\partial t^2} \quad (1)$$

where

$$(P, I) = \int_z \rho(1, z^2)dz \quad (2)$$

The displacement components based on YNS theory [1] are assumed to be

$$u = u_0(x,y,t) + z\psi_x(x,y,t)$$

$$v = v_0(x,y,t) + z\psi_y(x,y,t)$$

$$w = w(x,y,t) \quad (3)$$

where u, v and w are the displacement components in the x, y and z directions respectively, u_0, v_0 and w are the in-plane displacements of the middle plane and ψ_x and ψ_y are the shear rotations of any point on the middle surface of the plate.

The strain–displacement relations of linear elasticity may be written as

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + z\frac{\partial \psi_x}{\partial x}$$

$$\varepsilon_y = \frac{\partial v_0}{\partial y} + z\frac{\partial \psi_y}{\partial y}$$

$$\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right)$$

$$\gamma_{xz} = \psi_x + \frac{\partial w}{\partial x}$$

$$\gamma_{yz} = \psi_y + \frac{\partial w}{\partial y} \quad (4)$$

The stress–strain relations for the kth layer, after neglecting transverse normal strain and stress, are of the form

$$\begin{pmatrix} \sigma_x^{(k)} \\ \sigma_y^{(k)} \\ \tau_{xy}^{(k)} \\ \tau_{yz}^{(k)} \\ \tau_{xz}^{(k)} \end{pmatrix} = \begin{pmatrix} C_{11}^{(k)} & C_{12}^{(k)} & C_{16}^{(k)} & 0 & 0 \\ C_{12}^{(k)} & C_{22}^{(k)} & C_{26}^{(k)} & 0 & 0 \\ C_{16}^{(k)} & C_{26}^{(k)} & C_{66}^{(k)} & 0 & 0 \\ 0 & 0 & 0 & C_{44}^{(k)} & C_{45}^{(k)} \\ 0 & 0 & 0 & C_{45}^{(k)} & C_{55}^{(k)} \end{pmatrix} \begin{pmatrix} \varepsilon_x^{(k)} \\ \varepsilon_y^{(k)} \\ \gamma_{xy}^{(k)} \\ \gamma_{yz}^{(k)} \\ \gamma_{xz}^{(k)} \end{pmatrix} \quad (5)$$

where $Q_{ij}^{(k)}$, as functions of $C_{ij}^{(k)}$ and θ, are fully furnished in [59].

The stress resultants and stress couples are given by

$$(N_x, N_y, N_{xy}, Q_x, Q_y) = \int_z (\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz})dz$$

$$(M_x, M_y, M_{xy}) = \int_z (\sigma_x, \sigma_y, \tau_{xy})zdz \quad (6)$$