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# Laminar natural convection of power-law fluids in a square enclosure submitted from below to a uniform heat flux density

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Two-dimensional steady-state simulations of laminar natural convection of non-Newtonian power-law fluids in square enclosures heated through the lower horizontal wall have been carried out for constant wall heat flux boundary conditions. The effects of power-law index $n$ on heat and momentum transport have been analysed for nominal values of Rayleigh number ($Ra$) in the range $10^3$–$10^5$ and a Prandtl number ($Pr$) range of 10–$10^6$. It has been demonstrated that the mean Nusselt number $\overline{Nu}$ increases with increasing values of Rayleigh number for both Newtonian and power-law fluids. Moreover, $\overline{Nu}$ values obtained for power fluids with $n < 1$ ($n > 1$) are greater (smaller) than that obtained in the case of Newtonian fluids with the same nominal Rayleigh number $Ra$ due to strengthening (weakening) of convective transport. The effects of convection strengthen with increasing $Ra$ for a given set of values of $Pr$ and $n$, which is reflected in the increasing trend of $\overline{Nu}$ with increasing $Ra$. By contrast, the Prandtl number is shown to have marginal influence on $\overline{Nu}$. In addition a detailed comparison has been undertaken between these new results for the case of heating from below with existing results for the sidewall heating case. It has been found that $\overline{Nu}$ in the differentially heated horizontal wall configuration assumes smaller values than in the differentially heated vertical wall configuration for a given set of values of $n$ and Prandtl number in shear-thinning fluids (i.e. $n < 1$) for high values of $Ra$, whereas $\overline{Nu}$ values remain comparable for both differentially heated vertical and horizontal wall configurations for the Newtonian (i.e. $n = 1$) and shear-thickening fluids (i.e. $n > 1$). However for small values of Rayleigh number, $\overline{Nu}$ attains greater values in the differentially heated horizontal wall configuration for Newtonian ($n = 1.0$) and shear-thinning ($n < 1$) fluids. In contrast, $\overline{Nu}$ assumes higher values in the differentially heated vertical sidewall configuration for shear-thickening fluids($n > 1$) for small values of $Ra$. Detailed physical explanations have been provided for the observed $Ra$, $Pr$ and $n$ dependences of $\overline{Nu}$. A new correlation has been proposed for $\overline{Nu}$ for natural convection of power-law fluids in square enclosures heated from below subjected to constant heat fluxes. The new correlation is shown to satisfactorily capture both the qualitative and quantitative behaviour of $\overline{Nu}$ in response to the changes in $Ra$, $Pr$ and $n$ obtained from simulation data.



## 1. Introduction

Natural convection in rectangular enclosures has been analysed extensively for Newtonian fluids [1–4]. Several configurations can be possible for natural convection in rectangular enclosures based on the exact choice of boundary conditions used for the enclosure walls [4]. One of the most well-known configurations involves differentially-heated horizontal walls with the hot wall at the bottom while the vertical sidewalls are kept adiabatic, which is often referred to as the "Rayleigh–Bénard" configuration when the separation between the vertical walls is sufficiently wide [5]. In the current paper we are interested in precisely these thermal boundary conditions in a square enclosure (i.e. a rectangular enclosure of aspect ratio equal to unity). In comparison to a rather well developed and significant literature on natural convection of Newtonian fluids (consult Ref. [4] and references therein for example), relatively limited effort has been directed to the analysis of natural convection of non-Newtonian fluids. However natural convection in rectangular enclosures with adiabatic vertical walls and differentially heated horizontal walls with the bottom wall at higher temperature has been investigated for a limited range of different non-Newtonian models including inelastic Generalised Newtonian Fluids (GNFs) [6–10], yield stress fluids [11–15] and viscoelastic fluids [16].

Lamsaadi et al. [7] analysed the effects of nominal Rayleigh number $Ra$, Prandtl number $Pr$ and power-law index $n$ on natural

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**Nomenclature**

| | |
|---|---|
| $c_p$ | specific heat at constant pressure (J/kg K) |
| $e$ | relative error (–) |
| $e_{ij}$ | rate of strain tensor ($s^{-1}$) |
| $g$ | gravitational acceleration ($m/s^2$) |
| $Gr$ | Grashof number (–) |
| $h$ | heat transfer coefficient ($W/m^2$ K) |
| $K$ | consistency ($N s^n/m^2$) |
| $k$ | thermal conductivity (W/mK) |
| $L$ | length and height of the enclosure (m) |
| $n$ | power-law index (–) |
| $Nu$ | local Nusselt number (–) |
| $\overline{Nu}$ | mean Nusselt number (–) |
| $Pr$ | Prandtl number (–) |
| $q$ | heat flux ($W/m^2$) |
| $r$ | ratio between the coarse to fine grid spacings (–) |
| $r_e$ | grid expansion ratio (–) |
| $Ra$ | Rayleigh number (–) |
| $T$ | temperature (K) |
| $t$ | time (s) |
| $u_i$ | $i$th velocity component (m/s) |
| $U$, $V$ | dimensionless horizontal ($U = u_1L/\alpha$) and vertical velocity ($V = u_2L/\alpha$) (–) |
| $\vartheta$ | characteristic velocity (m/s) |
| $x_i$ | coordinate in $i$th direction (m) |
| $\alpha$ | thermal diffusivity ($m^2/s$) |
| $\beta$ | coefficient of thermal expansion (1/K) |
| $\delta$, $\delta_{th}$ | velocity and thermal boundary layer thickness (m) |
| $\theta$ | dimensionless temperature ($\theta = (T - T_{cen})k/(qL)$) (–) |
| $\mu_a$ | apparent dynamic viscosity ($N s/m^2$) |
| $\mu_{eff}$ | effective viscosity ($N s/m^2$) |
| $\nu$ | kinematic viscosity ($m^2/s$) |
| $\rho$ | density ($kg/m^3$) |
| $\tau_{ij}(\tau)$ | stress tensor (stress) (Pa) |
| $\phi$ | general primitive variable (–) |
| $\psi$ | dimensional stream function ($m^2/s$) |
| $\Psi$ | dimensionless stream function (–) |

*Subscripts*

| | |
|---|---|
| $C$ | cold wall |
| $cen$ | geometrical centre of domain |
| $ext$ | extrapolated value |
| $eff$ | effective value |
| $H$ | hot wall |
| $HW$ | horizontal wall configuration |
| $max$ | maximum value |
| $ref$ | reference value |
| $SW$ | vertical sidewall configuration |
| $wall$ | wall value |

*Special characters*

| | |
|---|---|
| $r$ | grid expansion ratio (–) |
| $\Delta T$ | difference between hot and cold wall temperature (K) |
| $\Delta_{min,cell}$ | minimum cell distance (m) |

convection of power-law fluids in shallow rectangular enclosures with differentially-heated horizontal walls subjected to constant heat flux[1]. The analytical and numerical results by Lamsaadi et al. [7] indicated that the mean Nusselt number increases with decreasing (increasing) power-law index $n$ (nominal Rayleigh number $Ra$). However, the nominal Prandtl number $Pr$ was found to have no major influence on the mean Nusselt number for the ranges of Rayleigh number and power-law index considered by Lamsaadi et al. [7]. Recently, Alloui et al. [8] analysed the onset of fluid motion for power-law fluids in the configuration previously studied by Lamsaadi et al. [7] using both analytical and numerical methods. They elegantly showed that convection in the enclosure can, in fact, occur for any non-zero value of Rayleigh number for $n > 1$ (i.e. the onset of convection is supercritical with a critical $Ra \to 0$), whereas convection sets in only once a critical Rayleigh number is surpassed in the case of shear-thinning fluids as is also observed for Newtonian fluids.

Ohta et al. [9] also reported augmentation of natural convection with increasing extent of shear-thinning based on a transient analysis of natural convection of shear-thinning fluids in rectangular enclosures with differentially-heated horizontal walls and adiabatic side walls. This finding is consistent with both experimental and numerical studies on micro-emulsion slurries by Inaba et al. [10] in the same configuration. Lamsaadi et al. [17,18] also analysed the effects of the power-law index on natural convection in the high Prandtl number limit for both tall [17] and shallow enclosures [18], where the vertical side walls boundary were subjected to constant heat fluxes. Lamsaadi et al. [17,18] showed that the convective heat transfer rate becomes dependent only on the nominal Rayleigh number $Ra$ and the power-law index $n$ for large values of aspect ratio and the nominal Prandtl number $Pr$. The same qualitative behaviour was reported by Kim et al. [19] and Turan et al. [20,21] for natural convection of power-law fluids in square enclosures with differentially-heated vertical side walls for both constant wall temperature [19,20] and constant wall heat flux [21] boundary conditions. Turan et al. [20,21] also proposed correlations for mean Nusselt number $\overline{Nu}$ for natural convection of power-law fluids in square enclosures with differentially heated vertical side walls for both constant wall temperature and constant heat flux boundary conditions. Recently, Turan et al. [22] analysed the effects of aspect ratio (i.e. ratio of enclosure height to length) on natural convection of power-law fluids in rectangular enclosures with differentially-heated vertical sidewalls subjected to both constant wall temperatures and heat fluxes. Barth and Carey [23] utilised more realistic GNF models which incorporate limiting viscosities at low and high shear rates to study a three-dimensional version of natural convection in square enclosures with differentially heated horizontal walls (the adiabatic boundary conditions for the side walls are replaced by a linear variation in temperature to match the experimental conditions of [24]). The effects of yield stress, Rayleigh and Prandtl numbers on natural convection of Bingham fluids have recently been addressed by Turan et al. [15,16] for square enclosures with differentially heated horizontal walls and adiabatic side walls. In the present work the analysis of Turan et al. [16] has been extended to numerically analyse the effects of $Ra$, $Pr$ and $n$ on the mean Nusselt number $\overline{Nu}$ for natural convection of Ostwald–De Waele (i.e. power-law) fluids in a square enclosure with differentially-heated horizontal walls subjected to constant wall heat flux heated from below. It is worth noting that most analyses on natural convection in rectangular enclosures with differentially-heated horizontal walls have been carried out for constant wall temperature boundary condition. By contrast, the same configuration with constant wall heat flux boundary condition for horizontal walls has rarely been analysed in the exiting literature. Recently, Kaddiri et al. [25] analysed the effects of temperature dependence of consistency $K$ on natural convection of power-law fluids in square enclosures with differentially-heated

[1] The definitions of $Ra$ and $Pr$ are provided later in Section 2.