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Semi-analytical modeling of composite beams using the scaled boundary finite element method

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ABSTRACT

The scaled boundary finite element method (SBFEM) is a semi-analytical method in which only the boundary is discretized. The results on the boundary are scaled into the domain with respect to a scaling center which must be "visible" from the whole boundary. For beam-like problems the scaling center can be selected at infinity and only the cross-section is discretized.

The current work is devoted to the development of some SBFEM elements for thin-walled beams on the basis of the first order shear deformation theory. The beam sections are considered to be multilayered laminate plates with arbitrary layup. The arbitrary cross-section is discretized with beam-like elements of Timoshenko type. Using the virtual work principle gives a system of differential equations of a gyroscopic type. The solution is given using the matrix exponential function.

Knowing the deformation of the beam, the stresses, strains and curvatures can be calculated and a failure criterion can be used to assess the laminate.

The SBFEM has been tested and compared with a finite element model and it gives good results.

1. Introduction

Beams and beam-like structures are widely used in mechanical and civil engineering. Due to lightweight reasons these beams are often made of thin-walled sections. Recently, new materials like fiber-reinforced plastics and other composites are used which are typically made of layers of differently oriented plies. With the number and the stacking sequence of the plies and the orientation and the thickness of each ply there are many parameters, which can be adjusted during an optimization. Thus, an effective and reliable computational method is needed.

1.1. The scaled boundary finite element method

The scaled boundary finite element method (SBFEM) is such a method. It is a semi-analytical method where only the boundary is dicretized and an analytical solution is used within the body. It doesn't need a singular fundamental solution like the boundary element method (BEM) or a discretization of the whole body like the finite element method (FEM). So it has the benefits of both the FEM and the BEM without adopting the shortcomings.

Let us start with a beam of arbitrary cross-section. Then the SBFEM uses the following separation approach for the displacements u in the framework of linear elasticity. The function u_1 scales the displacements of the boundary into the body or, like in this work, it scales the displacements of the cross-section along the beam. The behavior along the boundary or the cross-section, respectively, is described by the function u_2 for which a finite element approach is used. Thus, we have

$$u(x, y, z) = u_1(x) \cdot u_2(y, z) \tag{1}$$

The coordinates y and z are on the cross-section (or the boundary) and x is directed along the beam axis (or into the body) which for a beam is depicted in Fig. 1.

Inserting this ansatz into the virtual work principle gives a differential equation system of gyroscopic type (beam-like problem) or of Euler-type (scaling center within the body). For both situations the solution is known.

In essence, this method is a discrete Kantorovic method, which has been previously used by other working groups, as summarized below.

1.2. Literature review

For an arbitrary 3-dimensional case [1] published the first work about SBFEM and [2] developed this method further. It was used to

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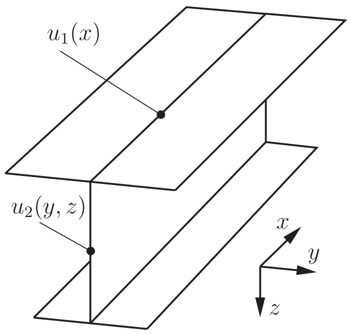

Fig. 1. SBFEM for a thin-walled beam.

calculate the dynamic stiffness of an unbounded domain. Originally this method was called "consistent infinitesimal finite-element cell method" but using a different way to derive the equations the name "scaled boundary finite element method" (SBFEM) has been introduced [3].

Due to the analytical part of the solution, the SBFEM can be used to calculate stress singularities at cracks, which is done in [3] and in [4].

In [5] thin plates under bending are described by using 1D elements for the boundary and Kirchhoff's kinematics are used to reduce the dimension by one.

For the case of slender cylindrical bodies like beams several groups developed similar or identical methods, where often 2-dimensional elements are used for the discretization of the cross-section.

In [6] a method is derived by introducing warping functions of the cross-section. These functions are only dependent on x. In [7] a separation ansatz is used, which gives the same results. The aim of both works is to calculate stiffnesses for the beam which are used in multi-body simulations.

Under the name "semi analytical finite elements" ([8–10]) a similar method is used to find de-Saint-Venant solutions. This method is employed to examine end effects and transitional effects in prismatic beams in [11]. Eventually, [12] extends this method for the calculation of wave reflections on free ends of cylinders.

In [13] the SBFEM is used to calculate free-edge effects in laminates, again using 2D elements.

In order to keep the number of degrees of freedom as low as possible the current work again addresses 1-dimensional elments.

Other works, where the 1-dimensional elements have been used, in general, apply the Kirchhoff theory. In [14], for instance, it is used to describe the wave reflection for thin-walled cylinders.

Ref. [15] developed a "generalized beam theory" for thin-walled beams where again Kirchhoff theory is taken as framework, but with some stresses neglected. [16] extends this method to orthotropic materials.

In [17,18] a "generalized Vlasov theory" is developed. Neglecting some stresses gives the theory of [15] or Vlasov.

In [19] 1D elements are used for the discretization of the cross-section of thin-walled beams, but only shear stresses and normal stresses in the direction of the beam axis are considered. Based on this theory "semi-analytical finite elements" were developed.

In contrast to former works, the current work uses a first order shear deformation theory to include transverse shear stresses and to gain a better accuracy.

2. Theory

Let us start presenting the theory for one single element. The assembly of several elements is described later.

For each flat element new coordinates (ξ, η and ζ) are introduced as follows: The coordinate ξ is running along the beam axis (x-axis) whereas the coordinates η and ζ are in the cross-section (y, z plane). Using the length ℓ of the beam and the width b of the element (see Fig. 2) the new coordinates are defined by the normalization $\xi = x/\ell$ and $\eta = y/b$ thus reaching from 0 to 1. The coordinate $\zeta = z$ defines the plate normal.

Similar coordinates are introduced for a cylindrical element (see Fig. 2). Again ℓ is the length of the beam and φ^* is the angle along the element. The coordinate $\xi = x/\ell$ is defined in an analogous manner as in the case of the flat element. In the y, z plane polar coordinates r and φ are introduced. Also the normalized angle $\vartheta = \varphi/\varphi^*$ is introduced running between 0 and 1 inside the element. Eventually, the thickness coordinate ζ starts in the midsurface, pointing into radial direction and also defines the local normal.

2.1. Kinematics

The kinematics of a Reissner-Mindlin plate is presumed [20]. One reason for that is that the Reissner-Mindlin theory is of higher order than the Kirchhoff theory and includes transversal shear. Another reason is that despite the extended kinematics in the SBFEM an element with Reissner-Mindlin theory has less degrees of freedom than an element based on Kirchhoff's kinematics.

A simple element with Kirchhoff's kinematics has 8 degrees of freedom (dof) but gives an equation of 4th order. After linearization it has $8 \cdot 4 = 32$ dofs. The linear Reissner-Mindlin element on the other hand has 10 dofs, but the equation is only of 2nd order. Thus, it has $10 \cdot 2 = 20$ dofs.

The third reason is, that the corresponding equations are easier to solve. Using the Reissner-Mindlin kinematics gives a differential equation of second order, where the matrix, which is multiplied with the highest derivative, has a full rank. So the inverse can be calculated and it can be written as a system of differential equations of first order, which can be solved with the numerically stable matrix exponential function.

In contrast to this using Kirchhoff's kinematics gives a second order differential equation for the in-plane deformation and a fourth order differential equation for the out-of-plane deformation (the bending and torsion). This is why the matrix at the highest derivative has not a full rank, which makes it difficult to solve the differential equation system in a numerically stable way.

The key idea of Reissner-Mindlin theory is that the displacements are traced back to the displacements and rotations of the middle plane (index 0)

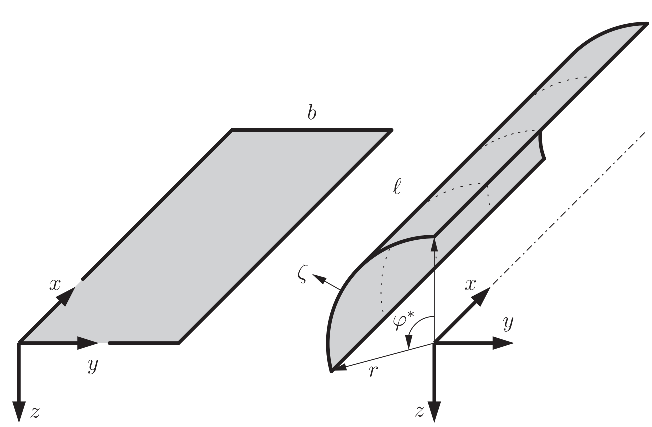

Fig. 2. SBFEM elements for flat and circular sections.