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Spectral stiffness microplane model for damage and fracture of textile composites

Marco Salviato, Shiva Esna Ashari, Gianluca Cusatis *

Department of Civil and Environmental Engineering, Northwestern University, Evanston, IL 60208, USA

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ABSTRACT

This contribution proposes a general constitutive model to simulate the orthotropic stiffness, pre-peak nonlinearity, failure envelopes, and the post-peak softening and fracture of textile composites.

Following the microplane model framework, the constitutive laws are formulated in terms of stress and strain vectors acting on planes of several orientations within the material meso-structure. The model exploits the spectral decomposition of the orthotropic stiffness tensor to define orthogonal strain modes at the microplane level. These are associated to the various constituents at the mesoscale and to the material response to different types of deformation. Strain-dependent constitutive equations are used to relate the microplane eigenstresses and eigenstrains while a variational principle is applied to relate the microplane stresses at the mesoscale to the continuum tensor at the macroscale.

The application of the model to a twill 2×2 shows that it can realistically predict its uniaxial as well as multi-axial behavior. Furthermore, the model shows excellent agreement with experiments on the axial crushing of composite tubes, this capability making it a valuable design tool for crashworthiness applications.

The formulation is computationally efficient, easy to calibrate and adaptable to other kinds of composite architectures such as 2D and 3D braids or 3D woven textiles.

1. Introduction

Thanks to their excellent specific mechanical performances and the recent developments in manufacturing technologies, the range of engineering applications of textile composites is continuously expanding. Current applications include land, marine and air transportation, wind and tidal energy production, and blast protection of civil infrastructures and vehicles [1–3]. However, in order to take advantage of the outstanding characteristics of these materials, design tools to simulate the orthotropic stiffness, pre-peak nonlinearity, failure envelopes, and the post-peak softening and fracture are quintessential.

Since the pioneering works by Ishikawa and Chou [4,5] and Ishikawa et al. [6], several formulations have been proposed, with varying degrees of success, to model the elastic properties of textile composites [7–13] and their failure mechanisms [14–20]. In general, however, these models stand on strength criteria to describe failure of the mesoscale constituents thus lacking completely of any description of the fracture mechanics involved. This is a serious deficiency being extensive intra-laminar cracking one of the main failure mechanisms in most applications of textile composites.

Modeling the fracturing behavior of textile composites, not only requires a fracture mechanics framework, it also urges the acknowledgment of their *quasi-brittle* character which highly affects the process of crack nucleation and growth. In facts, due to the complex mesostructure characterizing quasi-brittle materials (such as composites and nanocomposites, ceramics, rocks, sea ice, bio-materials and concrete, just to mention a few), the extent of the non-linear Fracture Process Zone (FPZ) occurring in the presence of a macrocrack is usually not negligible [21]. The stress field along the FPZ is nonuniform and decreases with crack opening gradually, due to discontinuous cracking, crack bridging by fibers, and frictional pullout of inhomogeneities. As a consequence, the fracturing behavior and, most importantly, the energetic size effect and the quasibrittleness effect associated with structure geometry, cannot be described by means of the classical Linear Elastic Fracture Mechanics (LEFM). To capture the effects of a finite FPZ size, the introduction in the formulation of a characteristic (finite) length scale of the material is necessary [21,22]. This is attempted in the present work.

* Corresponding author.
E-mail address: g-cusatis@northwestern.edu (G. Cusatis).

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Inspired by a recent theoretical framework for unidirectional composites by Cusatis et al. [23,24], this contribution aims at proposing a general constitutive model to simulate the damaging and fracturing behavior of textile composites. The formulation stands on the definition of strain-dependent constitutive laws in terms of stress and strain vectors acting on planes of several orientations within the material meso-structure. In this way, the model can easily capture various physical inelastic phenomena typical of fiber and textile composites such as: matrix microcracking, micro-delamination, crack bridging, pullout, and debonding.

Thanks to the coupling with the crack band model [25,26], the formulation is endowed with a characteristic length dependent on the strength and the fracture energy of the material. This is key to capture the intra-laminar size effect, a salient feature of composite structures. This aspect, too often overlooked in the literature on composites, is a determinant factor for damage tolerance design of large composite structures.

2. Theoretical framework

2.1. Microplane model

Inspired by the slip theory of plasticity pioneered by Taylor [27] and later refined by Batdorf and Budiansky [28], the microplane theory was originally developed to describe the softening damage of heterogeneous but statistically isotropic materials such as concrete and rocks [29,30]. Since its introduction in the early 1980s, the microplane model for concrete has evolved through 7 progressively improved versions labeled as M1 [29,30], M2 [31], M3 [32], M4 [33,34], M5 [35], M6 [36], M7 [37] and it has been recently adopted for the simulation of concrete at early age [38]. Microplane models have also been developed for other complex materials such as jointed rock [39], sand, clay, rigid foam, shape memory alloys, and unidirectional and textile composites [23,24,40–44]. A high order microplane model [45] was also derived recently on the basis of an underlying discrete model [46,47].

A key feature of the microplane model is that the constitutive laws are formulated in terms of the stress and strain vectors acting on a generic plane of any orientation within the material meso-structure, called the *microplane*. These planes can be conceived as the tangent planes of a unit sphere surrounding every point in the three-dimensional space (Fig. 1a). The microplane strain vectors are the projections of the macroscopic strain tensor, whereas the macroscopic stress tensor is related to the microplane stress vectors via the principle of virtual work. The adoption of vectors rather than tensors makes the approach conceptually clearer while the introduction of microplanes allows to inherently embed the effect of the mesostructure into the formulation.

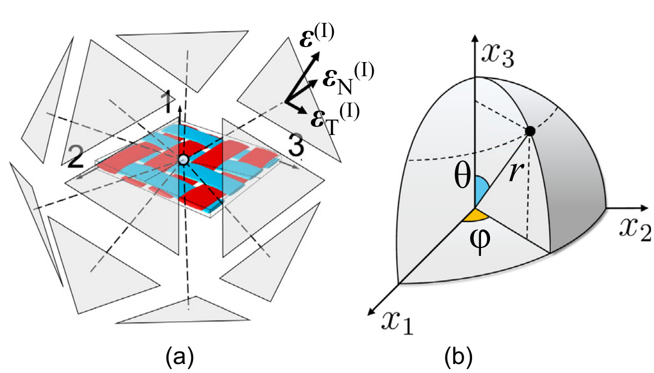

Fig. 1. Schematic representation of (a) the Representative Unit Cell of a 2 × 2 twill composite with its local coordinate system and the microplanes used to define the constitutive laws of the material; (b) local spherical coordinate system.

In this contribution, a kinematically constrained microplane model is adopted. This means that the strain vector on each microplane is the projection of the macroscopic strain tensor. In *kelvin notation* [48,49] this reads:

$$\boldsymbol{\varepsilon}_P = P\boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\varepsilon}_P = [\varepsilon_N\ \varepsilon_M\ \varepsilon_L]^T$ represents the microplane strain vector (Fig. 1a) with ε_N = normal strain component and ε_M and ε_L = shear strain components. Further,

$$P = \begin{bmatrix} N_{11} & N_{22} & N_{33} & \sqrt{2}N_{23} & \sqrt{2}N_{13} & \sqrt{2}N_{12} \\ M_{11} & M_{22} & M_{33} & \sqrt{2}M_{23} & \sqrt{2}M_{13} & \sqrt{2}M_{12} \\ L_{11} & L_{22} & L_{33} & \sqrt{2}L_{23} & \sqrt{2}L_{13} & \sqrt{2}L_{12} \end{bmatrix} \tag{2}$$

is a 3 × 6 matrix relating the macroscopic strain tensor to the microplane strain as a function of the plane orientation. As matter of fact, $N_{ij} = n_i n_j$, $M_{ij} = (m_i n_j + m_j n_i)/2$ and $L_{ij} = (l_i n_j + l_j n_i)/2$, where n_i, m_i and l_i are local Cartesian coordinate vectors on the generic microplane with n_i being the i-*th* component of the normal (Fig. 1a). With reference to the spherical coordinate system represented in Fig. 1b, the foregoing components can be expressed as a function of the spherical angles θ and $\varphi : n_1 = \sin\theta\cos\varphi,\ n_2 = \sin\theta\sin\varphi,\ \ n_3 = \cos\theta$ while one can choose $m_1 = \cos\theta\cos\varphi,\ m_2 = \cos\theta\sin\varphi,\ m_3 = -\sin\theta$ which gives, for orthogonality, $l_1 = -\sin\varphi$, $l_2 = \cos\varphi$ and $l_3 = 0$.

According to the microplane framework, the constitutive laws are then defined at the microplane level in a vectorial form. This makes the formulation conceptually clear and allows embedding the effect of the direction of damage in the constitutive law automatically. After the microplane stress vectors $\boldsymbol{\sigma}_P$ are computed, the macroscopic stress tensor is defined in a variational sense through the *principle of virtual work*:

$$\boldsymbol{\sigma} = \frac{3}{2\pi}\int_\Omega P^T \boldsymbol{\sigma}_P \mathrm{d}\Omega \tag{3}$$

where Ω is the surface of a unit sphere representing all the possible microplane orientations.

2.2. Spectral decomposition of the elastic tensor

In the microplane formulation, the material anisotropy is addressed by decomposing the stress and strain tensors into energetically orthogonal modes through the *spectral stiffness decomposition theorem* [50–53]. The following sections are intended to provide a brief introduction of the theory.

2.2.1. Spectral decomposition of the elastic tensor

The elastic behavior of a general anisotropic material can be expressed in *Kelvin notation* [48,49] as:

$$\boldsymbol{\sigma} = C\boldsymbol{\varepsilon} \tag{4}$$

where $\boldsymbol{\sigma} = \left[\sigma_{11}\ \sigma_{22}\ \sigma_{33}\ \sqrt{2}\sigma_{23}\ \sqrt{2}\sigma_{13}\ \sqrt{2}\sigma_{12}\right]^T$, $\boldsymbol{\varepsilon} = \left[\varepsilon_{11}\ \varepsilon_{22}\ \varepsilon_{33}\ \sqrt{2}\varepsilon_{23}\ \sqrt{2}\varepsilon_{13}\ \sqrt{2}\varepsilon_{12}\right]^T$ are the contracted forms of the stress and strain second-order tensors and C represents the contracted form of the fourth-order elastic tensor. The indices refer to Cartesian coordinates $x_i\,(i = 1, 2, 3)$ as defined in (Fig. 1a and b). It is worth mentioning here that the factor $\sqrt{2}$ assures that both the stiffness tensor and its column matrix have the same norm, given by the sum of the squares of their elements.

According to the spectral decomposition theorem [50–53], the stiffness matrix C can be decomposed as follows:

$$C = \sum_I \lambda^{(I)} C^{(I)} \tag{5}$$