Composite Structures 136 (2016) 217–228

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Nonlinear buckling of fibre-reinforced unit cells of lattice materials

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ARTICLE INFO

Article history:
Available online 10 October 2015

Keywords:
Mode interaction
Analytical modelling
Lattice materials
Structural stability
Nonlinear mechanics

ABSTRACT

Truss-based lattice materials are cellular materials with an outstanding potential for multi-functional use. This is owing to properties of high compressive strength to density ratios combined with a periodic and open structure. However, such structures at low relative densities are particularly vulnerable to elastic buckling failure. Fibre-reinforcement that increases the buckling strength of lattice materials is proposed and the behaviour of unit cells that are tessellated within the lattice is investigated. A two-dimensional square orientated unit cell and a three-dimensional tetrahedron-shaped unit cell are both modelled discretely using energy principles with the nonlinear interactive buckling behaviour being analysed. The analytical approach, based on a perturbation method, exhibits excellent agreement for the mechanical response when compared to results from numerical continuation for moderately large displacements. A fundamental understanding of the mechanical behaviour of a unit cell can be upscaled in future work. It is postulated that this will enable the determination of the constitutive behaviour of such lattice materials.

1. Introduction

In the course of the last few decades, cellular materials, for example metallic foams or honeycombs, have emerged in various engineering applications, such as core materials in sandwich panels, due to the combination of advantageous mechanical, thermal and acoustic properties [1]. Recent progress in additive manufacturing has enabled the production of truss-based lattice materials that, according to [2], outperform other cellular materials particularly in terms of the strength-to-density ratio. Lattice materials tend to have periodic, open topologies that offer possibilities of combining structural functions with thermal application, as shown in [3], or in medical applications [4]. However, the compressive strength of such materials may be limited by the collapse mechanism of the respective unit cell from which the lattice material is composed. Cellular materials comprising slender trusses, and hence low relative densities, are known to be prone to elastic buckling of their internal structure [5]. In the current context, relative density $\bar{\rho}$ is defined by the expression:

$$\bar{\rho} = \frac{\rho^*}{\rho_s} \tag{1}$$

and refers to the ratio of the densities of the actual cellular material ρ^* to that of a solid body made from the parent material ρ_s. The correlation between low relative densities and an increased vulnerability to elastic buckling was demonstrated experimentally for different geometries and materials in [6] for $\bar{\rho} = 0.03$ and [7] for $\bar{\rho} = 0.014$. A mechanism for increasing the strength in axial compression that is widely applied in civil engineering is to reinforce compression members by introducing pretensioned elements such as cable stays. This leads to a higher buckling resistance as the cables help to restrain the structure against the initial displacement during buckling [8–13]. By transferring this concept towards lattice materials the maximum compressive strength can be potentially increased beyond the conventional eigenvalue buckling load while avoiding a considerable gain in self-weight, or in the current case avoiding a significant increase in $\bar{\rho}$.

The objective of the current work is to investigate the potential effect of interwoven fibres on the critical and post-buckling response of lattice materials. Therefore, a fibre-reinforced lattice material is proposed based on an existing square orientated lattice material discussed in [6]. The deformational behaviour of the unit cell in the internal structure under axial compression is investigated using an analytical approach focusing on elastic buckling behaviour in the nonlinear range. Discrete models of unit cells comprising rigid links and springs, initially in two-dimensions and subsequently in three-dimensions are formulated using total potential energy principles. The performance of each model is eval-

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http://dx.doi.org/10.1016/j.compstruct.2015.09.059
0263-8223/

uated in terms of the critical and post-buckling behaviour both analytically and numerically. Potentially important nonlinear interactions between different instability modes in the post-buckling range and the consequences to the overall stability are investigated. It has been demonstrated previously that a fundamental understanding of system behaviour can facilitate exploitation of these lightweight materials with safety [14]. The article concludes with a discussion where some detailed suggestions for further work are made.

2. Fibre-reinforced square orientated lattice material

2.1. Material development

Fig. 1 shows a square orientated lattice material suggested and experimentally evaluated with respect to its out-of-plane compressive behaviour in [6]. Fig. 1(b) emphasises the vulnerability to elastic buckling failure of the internal structure with low relative densities. This failure mechanism may be suppressed by a square lattice structure that is reinforced by fibres. The fibres contribute to the buckling resistance and hence to the overall compressive strength of the lattice material. A computer aided design (CAD) model of such a fibre-reinforced lattice material is presented in Fig. 2.

2.2. Model development

A unit cell of the internal fibre-reinforced structure is modelled to obtain a deeper understanding of the deformational behaviour at cell level during elastic buckling. Therefore, a cross-shaped unit cell, as highlighted in Fig. 2(b), is modelled as a multiple-degree-of-freedom system in two dimensions, as represented in Fig. 3. The horizontal and vertical struts are modelled by pin-jointed rigid links, whereas the joints are reinforced by rotational springs of stiffness c_y. The horizontal strut is connected to the vertical strut at mid-height rigidly. Moreover, a lateral spring acts with stiffness k at mid-height of the horizontal strut to model the lateral resistance contributed by the interwoven fibres. The cell is assumed to be fully fixed at the upper and lower ends. Hence, the x-displacement and rotation of the outer links of length a is prevented such that they remain vertical. The respective ends of the horizontal crossarm, assigned with length b, are allowed to move in the x- as well as the z-direction. However, it is assumed that they remain horizontal as well as at the same height as the rigid corner. This enables the comprehensive description of the deformational behaviour at the cell level using only two generalised coordinates q_1, q_2. In the case where one cell would be embedded into a grid of many cells, the constraints arising from the neighbouring cells would need to be considered also.

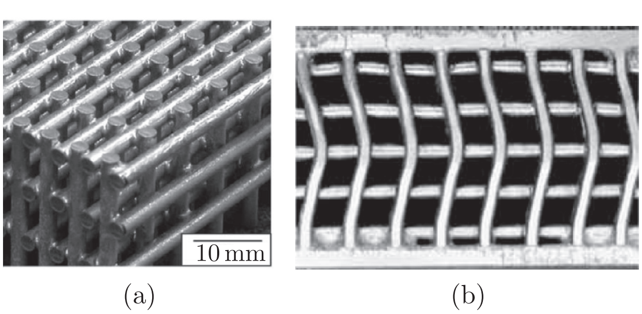

Fig. 1. Photographs of a square orientated lattice material showing (a) the geometry and (b) the deformational behaviour exhibiting fibre buckling within the internal structure [6].

2.3. Total potential energy

The total potential energy function V for a single cell is formulated by evaluating the total strain energy stored in all the springs U and the work done by the external load $P\Delta$ [15]:

$$V(q_i, P) = U(q_i) - P\Delta(q_i) \tag{2}$$

where currently $i = \{1, 2\}$. The total strain energy is decomposed into two constituent parts, the contribution from the longitudinal springs U_L and those from the rotational springs U_R. These terms are developed by considering an arbitrarily deflected state in the xz-plane, as shown in Fig. 3(b). The energy stored in the lateral spring gives the expression:

$$U_L = \frac{kl^2}{8}(q_1 + q_2)^2. \tag{3}$$

For the strain energy stored in the rotational springs, the behaviour of the horizontal strut becomes significant due to the rigid joint at mid-height. Hence, U_R consists of one component being active, *i.e.* non-zero, for every deflected shape and a second component being non-zero only for buckling shapes where $q_1 \neq q_2$. This leads to the expression:

$$U_R = \frac{c_y}{2}\Big[\theta_1^2 + \theta_3^2 + \theta_4^2 + \theta_6^2 + (\theta_1 - \theta_2)^2 + (\theta_2 + \theta_3)^2 + (\theta_4 - \theta_5)^2 + (\theta_5 + \theta_6)^2\Big], \tag{4}$$

in which:

$$\theta_1 = \arcsin(q_1), \quad \theta_2 = \arcsin(q_2 - q_1), \quad \theta_3 = \arcsin(q_2),$$
$$\theta_4 = \arcsin\left(\frac{q_1 - q_2}{2}\right), \quad \theta_5 = \arcsin(q_2 - q_1), \quad \theta_6 = \arcsin\left(\frac{q_2 - q_1}{2}\right).$$

The end-shortening displacement Δ contributes to the work done by the load P and is given by the expression:

$$\Delta = l\left[3 - \left(\sqrt{1 - q_1^2} + \sqrt{1 - (q_2 - q_1)^2} + \sqrt{1 - q_2^2}\right)\right]. \tag{5}$$

Assuming only moderately large deformations, the energy is expressed as a power series and is truncated after order four. The total potential energy V can be non-dimensionalized by dividing through the rotational stiffness c_y, thus:

$$\widetilde{V}(q_1, q_2, p) = \frac{11}{2}q_1^2 - 9q_1q_2 + \frac{13}{2}q_1^2q_2^2 - \frac{9}{2}q_1^3q_2 + \frac{11}{2}q_2^2 - \frac{9}{2}q_1q_2^3 + \frac{19}{12}(q_1^4 + q_2^4) + \frac{K}{8}(q_1^2 + q_2^2 + 2q_1q_2) - p\left(q_1^2 - q_1q_2 + \frac{3}{4}q_1^2q_2^2 - \frac{1}{2}q_1^3q_2 + q_2^2 - \frac{1}{2}q_1q_2^3 + \frac{1}{4}q_1^4 + \frac{1}{4}q_2^4\right), \tag{6}$$

with the now non-dimensionalized parameters given by:

$$\widetilde{V} = \frac{V}{c_y}, \quad K = \frac{kl^2}{c_y}, \quad p = \frac{Pl}{c_y}.$$

The potential energy expression is also diagonalized using the following transformation:

$$u_1 = q_1 + q_2, \quad u_2 = q_1 - q_2. \tag{7}$$

This results in the diagonalized non-dimensional total potential energy expression $A(u_1, u_2, p)$:

$$A(u_1, u_2, p) = \frac{1}{2}u_1^2 + 5u_2^2 + \frac{3}{8}u_1^2u_2^2 + \frac{1}{24}u_1^4 + \frac{7}{6}u_2^4 + \frac{K}{8}u_1^2 - \frac{p}{4}\left(u_1^2 + 3u_2^2 + \frac{3}{8}u_1^2u_2^2 + \frac{1}{16}u_1^4 + \frac{9}{16}u_2^4\right). \tag{8}$$