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Steady flow of power-law fluids in a 1:3 planar sudden expansion

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ABSTRACT

The laminar flow of inelastic non-Newtonian fluids, obeying the power-law model, through a planar sudden expansion with a 1:3 expansion ratio was investigated numerically using a finite volume method. A broad range of power-law indices in the range $0.2 \leqslant n \leqslant 4$ was considered. Shear-thinning, Newtonian and shear-thickening fluids are analyzed, with particular emphasis on the flow patterns and bifurcation phenomenon occurring at high Reynolds number laminar flows. The effect of the generalized Reynolds number (based on power-law index, n, and the inflow channel height, h) on the main vortex characteristics and Couette correction are examined in detail in the range $0.01 \leqslant Re_{gen} \leqslant 600$. Values for the critical generalized Reynolds number for the onset of steady flow asymmetry and the appearance of a third main vortex are also included. We found that the shear-thinning behavior increases the critical Re_{gen}, while shear-thickening has the opposite effect. Comparison with available literature and with predictions using a commercial software (Fluent® 6.3.26) are also presented and discussed. It was found that both results are in good agreement, and that our code is able to achieve converged solutions for a broad range of flow conditions, providing new benchmark quality data.

1. Introduction

When a Newtonian fluid flows at low to moderate Reynolds number in a 2D planar channel and encounters a sudden expansion, flow separation occurs resulting in a pair of symmetric recirculating eddies along the downstream walls. The vortices become asymmetric, but steady, when the Reynolds number (Re) is increased above a certain critical value. With a further increase in Re a third vortex is formed downstream of the smallest of the two main vortices [1]. A bifurcation phenomenon, consisting of a transition from symmetric to asymmetric flow, occurs above a critical Reynolds number that depends on the expansion ratio of the planar expansion and the rheology of the fluid. The expansion ratio for a planar geometry is defined as the ratio of the height of the outlet channel (H) to the height of the inlet channel (h) and henceforth is denoted as ER.

Since the early 1970s there has been a number of experimental studies devoted to the subject of flow bifurcation in channels with a sudden planar expansion. Using laser Doppler anemometry (LDA) Durst et al. [1] examined the Newtonian fluid flow in a 1:3 planar symmetric expansion. In their experiments, two symmetric vortices along the walls of the expansion were observed at $Re = 56$. At $Re = 114$, flow bifurcation was already observed with vortices of unequal size forming at both salient corners. The experimental measurements of Cherdron et al. [2] also relied on LDA, but were more comprehensive and explored the flow patterns and instabilities in ducts with symmetric expansions, investigating also the effect of the aspect ratio of the tested geometries. The more recent experimental and numerical study of Fearn et al. [3] in a 1:3 planar expansion showed a similar flow bifurcation at a Reynolds number of 40.5. In contrast to the few experimental investigations, there is a large number of numerical works available in the literature and one of its advantages is that it is possible to investigate truly 2D flows. In his numerical investigation on planar expansion flows with various expansion ratios, Drikakis [4] found that the critical Reynolds number for the symmetry-breaking bifurcation is reduced when the expansion ratio is increased. Battaglia and Papadopoulos [5] studied the influence of three-dimensional effects on the bifurcation characteristics at low Reynolds number flows in rectangular sudden expansions, in the range of $150 \leqslant Re \leqslant 600$. All these experimental and numerical studies were concerned with Newtonian fluids.

In many realistic situations the fluids flowing through flow devices are non-Newtonian and show complex rheological behavior.

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Specifically, they can exhibit shear-thinning or shear-thickening viscosity depending on the type of fluid and thus it is relevant to investigate the non-Newtonian fluid flow in planar expansions starting with simple rheological models in order to independently assess the impact of specific rheological features upon the flow characteristics. If the non-Newtonian solutions are not too concentrated the flows tend to have a high Reynolds number, even leading to turbulent flow. Since the sudden expansion is a well-known geometry for studies of laminar flow instabilities at high Reynolds numbers, in recent years it has naturally started to attract the attention of researchers in the field of non-Newtonian fluid mechanics wishing to investigate the complex interaction between these bifurcations and fluid rheology, namely viscoelasticity. In non-Newtonian fluid mechanics there are other traditional benchmark flows, such as the 4:1 sudden contraction and the flow around a confined cylinder under 50% blockage ratio, but these have been devised to address the numerical convergence difficulties in creeping flows of viscoelastic fluids. As we show below, the investigations of power law fluids carried out so far in a 1:3 planar expansion provide an incomplete picture, which we aim to address and complete in this work. It is to be noted that flow of blood (which is non-Newtonian) in arterial stenoses and abdominal aneurysms are relevant to flow in expansions.

The non-Newtonian power-law model is the simplest model for a purely viscous fluid that can represent the behavior of shear-shinning, shear-thickening and Newtonian fluids by varying the parameter of the model, n, known as the power-law index. Consequently, it comes as no surprise that several numerical studies in the past were performed using the power-law viscosity model to study the flow of shear-thinning and shear-thickening fluids in planar sudden expansions of various ER.

Mishra and Jayaraman [6] examined numerically and experimentally the asymmetric steady flow patterns of shear-thinning fluids through planar sudden expansions with a large expansion ratio, $ER = 16$. Manica and De Bortoli [7] studied numerically the flow of power-law fluids in a 1:3 planar sudden expansion for $n = 0.5$, 1 and 1.5. They presented the vortex characteristics for these values of n and for $30 \leqslant Re \leqslant 125$, and observed that the flow bifurcation for shear-thinning fluids occurs at a critical Reynolds number higher than for Newtonian fluids, and that shear-thickening fluids exhibited the lowest critical Reynolds number. Considering again purely viscous fluids represented by the power-law and Casson models, Neofytou [8] analyzed the transition from symmetric to asymmetric flow of power-law fluids with power-law indices in the range $0.3 \leqslant n \leqslant 3$ in a 1:2 planar sudden expansion and also studied the effect of Reynolds number on the flow patterns.

Ternik et al. [9] studied the flow through a 1:3 planar symmetric expansion of non-Newtonian fluids with shear-thickening behavior using the quadratic and power-law viscosity models. They compared the results of both models with those of Newtonian fluids and concluded that the occurrence of flow asymmetry is greatly affected by the shear-thickening behavior. Later, Ternik [10], computed the flow of shear-thinning fluids with power-law indices $n = 0.6$ and 0.8 in a 1:3 planar sudden expansion. After the first bifurcation, from a symmetric to asymmetric flow, a second flow bifurcation, marking the appearance of a third vortex, was predicted as the generalized Reynolds number was further increased, with shear-thinning delaying the onset of this second bifurcation. More recently, Ternik [11] revisited the generalized Newtonian flow in a two-dimensional 1:3 sudden expansion using the open source OpenFOAM CFD software. The fluid was again represented by the power-law model with power-law index in the range $0.6 \leqslant n \leqslant 1.4$ and the simulations were performed for generalized Reynolds numbers in the range $10^{-4} \leqslant Re_{gen} \leqslant 10$ with the emphasis on the analysis of low Reynolds number flows, below the critical conditions for the onset of the pitchfork bifurcation.

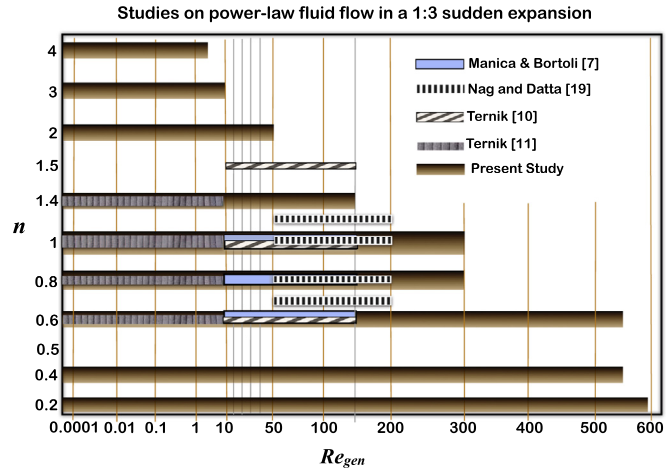

Fig. 1. Graphical illustration of the range of generalised Reynolds numbers (Re_{gen}) and power-law index (n) used in various studies in the archival literature. (See above-mentioned references for further information.)

Small recirculations, typical of creeping flow (called Moffatt vortices [12]) were observed for all fluids with shear-thinning behavior reducing the size and intensity of the secondary flow.

Numerical simulations of the flow of power-law fluids in a planar 1:3 sudden expansion using commercial or open source codes were attempted by several authors. It was found that the solution convergence is often a major limitation when utilizing these codes especially when the non-Newtonian behavior is enhanced (large or small n values for power-law model). For instance, Poole and Ridley [13] used Fluent® software to numerically calculate the development-length required to attain fully developed laminar pipe flow of inelastic power-law fluids and were unable to attain a converged solution for $n < 0.4$. Ternik [10] reported that the iterative convergence had become increasingly time consuming with a reduction in power-law index, and for $n < 0.6$ no converged solutions were obtained using the OpenFOAM software.

From the aforementioned discussion, it is clear that a comprehensive investigation on the flow of power-law fluids in planar sudden expansions is still lacking for power-law indices below $n = 0.5$ and above $n = 1.5$ and this is clearly seen in Fig. 1. This work aims to fill this gap in the literature using an in-house finite-volume code [14]. We present a systematic study of the flow in a 1:3 sudden planar expansion for a wide range of power-law indices, $0.2 \leqslant n \leqslant 4$, and generalized Reynolds numbers, $0.01 \leqslant Re_{gen} \leqslant 600$, including data for the Couette correction. The critical generalized Reynolds number at which symmetry breaking flow bifurcation occurs is reported and the flow structures in the expansion are visualized using streamline plots. We also compare the results obtained with our in-house code with those calculated using the Fluent® 6.3.26 software using exactly the same meshes and flow conditions. The remainder of this paper is organized as follows: in Section 2 we present the mathematical formulation of the problem, and in Section 3 we discuss the numerical method along with the code validation. The results are presented and discussed in Section 4, and Section 5 summarizes the main conclusions.

2. Mathematical formulation

2.1. Problem description

The problem under study is illustrated schematically in Fig. 2a, which also includes the nomenclature used to refer to the various characteristic lengths of the vortices. A 2D, long, planar channel of width h has a sudden expansion to a second channel of width H,