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# Application of lubrication theory to modeling stack geometry of high density mine tailings

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ABSTRACT

Consideration of non-Newtonian flow behavior has been important in the management of mineral resource wastes for some time, chiefly in regards to dewatering and pumping of concentrated suspensions of mill tailings. These dewatered tailings, often referred to as "thickened" or "high-density" tailings, may be deposited on surface, and if they are sufficiently dewatered to exhibit a yield stress, they will form gently sloped stacks. The geometry of these stacks is in part influenced by the rheology of the tailings, as well as by depositional parameters such as flow rate and the overall slope of the stack. Predicting or managing the evolution of the geometry of such deposits or stacks of high density tailings, is an important issue in mine waste management – mine tailings impoundments can cover several square kilometers, therefore the average slope strongly affects the capacity of the tailings storage impoundment and therefore cost and environmental footprint. Controlling layer thickness is also important for operations where evaporation is relied up to further dewater the tailings. The use of non-Newtonian flow theory may be useful in anticipating the geometric evolution of such impoundments. This paper describes three-dimensional bench scale physical simulations of the deposition of high density tailings from a gold mine. The experimental results are modeled using lubrication theory based equations for equilibrium profiles of simple geometries. Using the yield stress interpreted from slump tests, the lubrication theory equations provide good estimates of the evolving stack, despite the sometimes asymmetric nature of the three-dimensional flows. Limited comparisons are made to a small-set of field data. The behavior of the laboratory stack shows good qualitative agreement with early deposition in the field. Two important issues are discussed, one, the tendency of the stack to evolve a convex profile characterized by high slopes near the deposition point and lower slopes in the later stages of deposition, and two, the influence of settling and capillary action on the rheological behavior of tailings while they are flowing.



## 1. Introduction

Mine tailings are the by-product of grinding and other physical and chemical processes employed in milling to extract a desired mineral from subsurface material. Tailings are comprised of solid particles, predominantly between 100 and 2 μm in diameter, suspended in water. A significant amount of water can be used in the various extraction processes, and the water content of the tailings that emerges from the mill is sufficiently high that the tailings generally exhibit no shear strength and therefore are conventionally deposited in dammed impoundments. Consider a gold mine that has a mill capacity of 200,000 tonnes of rock per day. Given the small fraction of the extracted mineral, virtually all the rock ends up as tailings. Also considering that conventionally deposited gold tailings typically have a solids concentration (Mass of solids/Mass of total) of 50%, 200,000 $m^3$ of water is deposited into the tailings daily. If one imagine a mine life of several years, it is easy to explain the size of typical impoundments, which cover several square kilometers and often contained by dams over a 100 m high. Such conventional disposition practices incur significant liabilities, including the cost of water or water treatment, seepage from tailings impoundments, and the risk of catastrophic dam failure unfortunately associated with conventional impoundments [1] – in one case the cleanup cost of one such failure was in the order of US $250 million. Fig. 1a shows a well-known example of the failure of a conventional tailings impoundment in Southern Spain.

Disposal of dewatered or "thickened" tailings is an alternative for surface disposal that eliminates or reduces some of the risks associated with conventional practice, most notably obviating the need for dams, increasing water recycling within the mining operation, and facilitating rapid reclamation. Recent advances in technology have permitted economic dewatering and pumping of thicker tailings as relatively concentrated suspension that manifest a yield stress, forming gently sloping stacks during deposition, thus reducing reliance on confinement by dams. Tailings thickened to

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(a)

(b)

**Fig. 1.** Failure of a dammed tailings impoundment (a) and early deposition at a thickened tailings impoundment (b).

the extent that no segregation of particle size occurs during transport and relatively small amount of settling occurs post-deposition are often called ‘‘paste’’ [2]. Thickened tailings may be deposited cyclically from a number of deposition points: this allows tailings to gain significant strength through desiccation and/or drainage before placement of subsequent layers. From a flow modeling point of view, the next to last layer may be treated as a stable surface that serves as the underlying boundary for the fresh layer. Thickened tailings deposition has been employed by a increasing percentage of mining operations in hard rock tailings [3], bauxite tailings [4], and recently, oil sand tailings [5,6]. Fig. 1b shows an example of surface deposition from a gold mine in a relatively arid environment. The popularity of thickened tailings deposition has been increasing as the cost and reliability of dewatering technologies improves and the advantages of thickened tailings technology have been more widely acknowledged in the industry. However, some uncertainty regarding the performance of thickened tailings impoundments does exist: specifically, in the areas of seismic stability of the stack, susceptibility to acid rock drainage (ARD), and what overall slope angle can be reliably achieved. The last can have very important cost implications as it dictates the required footprint of the facility, and/or the height of any perimeter embankments.

While the behavior of thickened or paste tailings in the pipeline have been studied to a significant degree [7–9], the relation between rheology and deposition geometry has received less attention. Most studies have focused on charactering the geometry using a single angle [10,4] at the laboratory scale. However, it is observed that the overall angles of deposits in the field are typically substantially less than angles measured in the laboratory [11,12]. This was partially thought to be attributed to shear thinning occurring during transport (For example, [13]). However, Simms [14] proposed, based on non-Newtonian flow theory that had been previously applied to mud and lava flows [15], that the equilibrium beach profile is not characterized by a unique angle, and that the overall angle of the deposit is a function the size of the flow. Henriquez and Simms [16] verified the applicability of lubrication theory to describe thickened tailings flows for a gold tailings at the laboratory, and observed the scale-dependency of the overall deposition angle of a single layer deposits. Others have tackled this problem using an approach based on upon the mechanics of channeling flow, where the equilibrium slope is taken to characterize the maximum slope beyond which the tailings will self-erode [17–19]. Others again have pursued an approach based upon dissipation of kinetic energy of the tailings stream as it travels down the slope of the stack [17,20,21]. The variety of theories reflects different possible states of the tailings: laminar or turbulent, supercritical or subcritical, spreading or channelized flow. For example, tailings may be observed to exit the pipe as a supercritical flow, which undergoes a hydraulic jump close to the deposition point, and subsequently converts into a spreading subcritical flow. On the same stack but later in the deposition process, the flow does not undergo a hydraulic jump, and rather forms a narrow channel, only spreading out near the bottom of the stack (Fig. 2).

Free surface flows of a non-Newtonian material over a varying topography is important to many applications such as avalanches, mud and debris flows. Consequently, these flows have received considerable attention. A review of recent literature is provided by [22]. Tailings behavior is complex, as it can exhibit both channelized flow and slow spreading behavior. The latter, as shown by [14,16,17], can be described by depth-averaged equations developed from lubrication theory. The former is considerably more challenging due to the eroding behavior. Johnson and Gray [23] investigates some of these more complex behaviors, though for flow of a dry granular material.

This paper focuses on and extends the work of [16] to multilayer and 3-dimensional deposits of thickened tailings at the laboratory scale. While [16] analyzed transient flow as equilibrium profiles of thickened tailings, the focus of this paper is to investigate the potential for change in the rheological properties of the tailings, while they are still flowing e.g, thixotropy. Subsequent sections introduce the depth-average lubrication theory equations, the material properties relevant to the rheology and dewatering behavior of the tailings, and then present static profiles obtained from bench-scale multilayer deposits, in flumes and in unconfined deposits. Finally, the findings of the laboratory program are compared to select field results.

## 2. Theory

Equations for the equilibrium profiles of yield stress fluids have been derived by several authors using ‘‘Lubrication Theory’’, in which the Navier–Stokes equations are simplified by assuming the slow spreading of a thin layer or film. The simplifying assumptions are

1. The ratio of thickness to horizontal extent of the flow is small.
2. The velocity of the material is slow, such that terms that include the ratio of inertial to viscous forces will vanish from the momentum equation.

These simplified momentum and continuity equations have been solved analytically for yield stress fluids under special geometries and special conditions by several researchers, for appli-