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A user-defined finite element for laminated glass panels and photovoltaic modules based on a layer-wise theory

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ABSTRACT

Laminated plates with glass skin layers and a core layer from polyvinyl butyral are widely used in the civil engineering and automotive industry. Crystalline or thin film photovoltaic modules are composed from front and back glass or polymer layers and a solar cell layer embedded in a polymeric encapsulant. For the structural analysis of such laminates, layer-wise theories (LWTs) for plates have been introduced in literature. In this paper, an extended LWT is proposed to assure the C_0 continuity. Based on this theory, a finite element formulation and a user-defined quadrilateral serendipity element with quadratic shape functions and nine degrees of freedom (DOFs) is presented. The element is implemented using the Abaqus subroutine "User Element". Benchmark problems are developed to examine the accuracy and efficiency of the proposed element for a wide range of material properties including the limiting cases of shear compliant and shear rigid laminates. An emphasis is placed on the influence of boundary conditions with respect to additional degrees of freedom as well as on accurate representation of boundary layer effects.

1. Introduction

In the civil engineering and automotive industry, laminated plates with glass skin layers and a core layer from polyvinyl butyral (PVB) are widely used [1–3]. Crystalline or thin film photovoltaic modules currently available on the market are composed from front and back glass or polymer layers and a solar cell layer embedded in a polymeric encapsulant [4–6]. A lightweight design of photovoltaic modules includes front and back panels made from plastics. These skin layers are connected together by a transparent polyurethane (PUR), in which the solar cells are embedded [7]. Fig. 1 illustrates different types of photovoltaic modules.

For design of glass laminates and photovoltaic modules it is beneficial to analyze the suitability of materials like PVB, ethylene–vinyl acetate (EVA), or PUR for embedding solar cells. These encapsulates have to compensate different mechanical and thermal strains of bottom and top layers. Delamination between the layers must be avoided and solar cells have to be protected against air and water. Mechanical properties of soft encapsulate materials are usually affected by the manufacturing process. Furthermore, environmental effects can lead to changes in the mechanical behavior over time. Therefore a reliable assessment of the stiffness properties is only possible by the testing of a prototype, e.g. by bending tests of a beam or a plate. To evaluate the test results, robust structural analysis methods are required to relate the global deformation state, for example the deflection, and the local stress and strain state, for example the transverse shear stresses and strains, to the applied external load.

One feature of laminated glass plates or laminates used in photovoltaic industry is the layered composite with relatively stiff skin layers and relatively thin and compliant polymer encapsulant layer. Let G_S be the shear modulus of the glass skin layer and G_C the shear modulus of the polymeric core layer. The ratio of the shear moduli $\mu = G_C/G_S$ for materials used in photovoltaics is in the range between 10^{-5} and 10^{-2}, depending on the type of polymer and the temperature [4,8,7]. For classical sandwich applications, this ratio is in the range of 10^{-2} and 10^{-1}. In addition, in classical sandwich structures the face sheets are thin in comparison to the core, while in photovoltaic applications the face layers are thick and the core is thin.

To analyze the behavior of laminated plates, various structural mechanics models are available. A widely used approach for sandwich and laminate structures is the first-order shear deformation theory (FSDT) [9,10]. The principal assumption of this theory is that any normal to the reference midplane of the plate behaves like a

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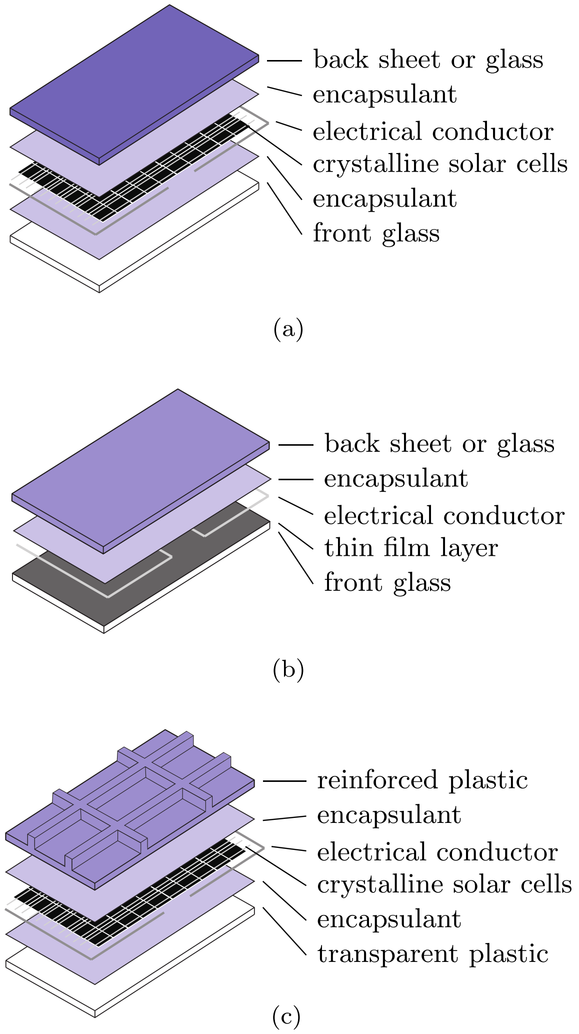

Fig. 1. Types of photovoltaic modules. (a) Crystalline photovoltaic module [4], (b) thin film photovoltaic module [4], (c) lightweight photovoltaic module [7].

rigid line during the deformation. The rotation degrees of freedom are independent from the transverse displacements, in contrast to the Kirchhoff theory, where the lines are assumed normal to the deformed midsurface. The advantage of this theory is the possibility to solve the governing differential equations in a closed analytical form for plates of various shapes. Closed-form or approximate analytical solutions for plates according to the FSDT are presented in [9–14], among others. Furthermore, different types of shell elements are available for the analysis of layered structures [15]. A problem related to FSDT is to estimate effective characteristics of the layered system, in particular the properties related to the transverse shear deformation. Closed-form relationships are developed to find effective elastic stiffness of a laminate from the properties of layers, e.g. [16,17]. However, numerical techniques are required to estimate the effective transverse shear deformation in the inelastic range [18,19]. It should be mentioned that for laminates with extreme differences in the stiffness of layers the FSDT fails to predict the deformation properties of the laminate correctly, as shown for example in [4,7] for beams and in [20,21] for plates.

Laminated glass and photovoltaic modules can also be analyzed using the three-dimensional theory of elasticity and applying solid finite elements for the numerical solution. However, due to extreme differences in material properties of constituents and the relatively low thickness of the core layer, considerable numerical effort is required to obtain results with a desired accuracy [4,22]. In particular, care should be taken for finite element meshing of the core layer in order to compute the transverse shear strains and the related stresses accurately.

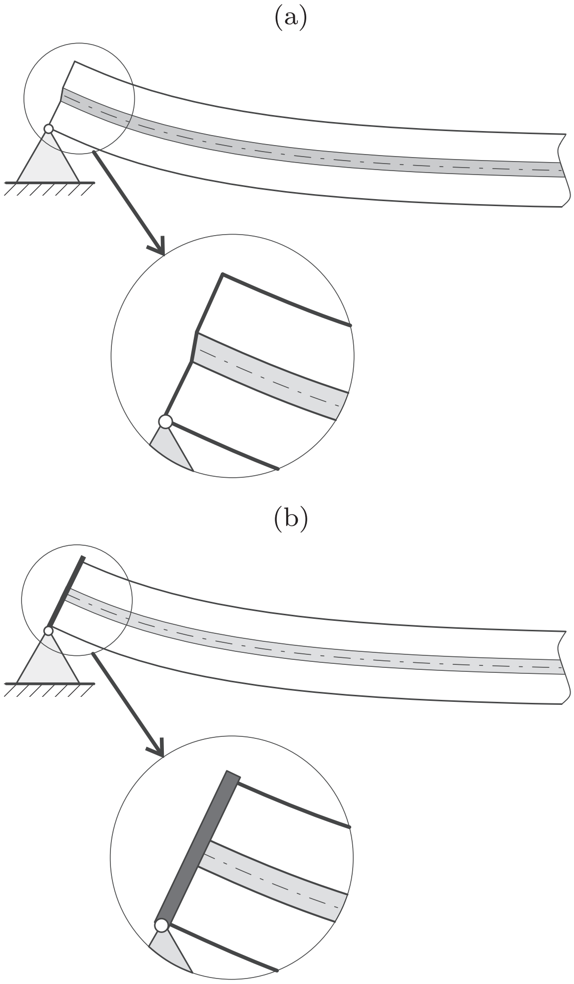

Fig. 2. Deformation of a plate edge for two types of supports. (a) Support without rotational constraint, (b) support with rotational constraint.

In addition to conventional shell and solid elements, continuum shell elements can also be applied to analyze laminates [23,24]. The finite element (FE) code Abaqus, for example, offers continuum shell elements, which possess only displacement DOFs and use three-dimensional constitutive equations [15]. They include the linear elements SC6R (triangular, 18 DOFs per element) and SC8R (quadrilateral, 24 DOFs per element). However, at least three elements in the thickness direction are required to analyze a three-layer laminate. Therefore the total number of DOFs required for analysis of an entire laminate can increase significantly. Furthermore, continuum shell elements with quadratic shape functions are not available in Abaqus.

In order to analyze laminated structures, zig-zag and layer-wise theories have been developed. A zig-zag theory approximates the displacements by piecewise functions with respect to the thickness coordinate such that the compatibility between the layers is fulfilled. Then the governing equations of the three-dimensional elasticity theory are reduced to the two-dimensional plate equations by means of variational methods or asymptotic techniques [25–27]. Within the layer-wise theory (LWT), equilibrium and constitutive equations are derived for individual layers. With constitutive assumptions for interaction forces and compatibility conditions, a model for the layered system is derived. For laminated