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# Geometrical non-linear analysis of fiber reinforced elastic solids considering debounding

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ABSTRACT

This study intends to contribute with the improvement of FEM formulations to analyze composite materials and structures at the meso and macro levels. Specifically, composite materials and structures constituted by an elastic matrix reinforced by long or short fibers that may debound when the contact stress reaches a critical value are taken into account.

The study proposes a way to introduce fibers (short or long) inside elastic solids modeled by finite elements without increasing the number of degrees of freedom when debounding is not present. When there is debounding, new degrees of freedom are incorporated only at the slipping contact between fibers and matrix following an elastoplastic non-linear constitutive relation. The matrix is considered elastic while fibers follow a multi-linear elastoplastic constitutive relation. Large deformations and moderate strains are considered. In order to model the debounding between fibers and matrix it is necessary to accurately calculate the contact stresses. Therefore, the development of straight and curved fiber elements of any order and the accuracy analysis of the proposed contact stress calculations are investigated. After defining the best strategy to be followed general examples are solved to validate the contact stress calculation, debounding and plastic fiber behavior.



## 1. Introduction

Composite materials are designed to combine two or more materials in order to take advantage of their better physical, mechanical or economic properties. This kind of material is largely employed in engineering solutions, from traditional applications as the reinforced concrete to more recent ones as fiber-carbon composites used in aeronautical industry.Regarding the combination of mechanical (and economical) properties we may cite some simple composite materials. The reinforced concrete takes advantage of the concrete low cost and its high compression strength and the ductility and high strength in tension of steel. The reinforced rubber takes advantage of the high deformability of rubber and its vibration absorbing properties and the high strength and stiffness of steel. In fiber-carbon composites the resin adherent properties are used together with the low weight, high strength and high stiffness of carbon-fibers to constitute light and tough structures.

The mechanical analysis of fiber-reinforced composites falls in three main levels: the macro-level, the meso-level and the nano-level. The first is interested in the overall behavior of structural components. The meso-level deals with the interdependent behavior of fiber and matrix, i.e. interfacial stress and debounding. Finally the nano-level is interested in the nano-scale constitution of fibers and matrix by themselves and in its influence at meso and macro-levels.

In this study we are interested to contribute with the improvement of FEM formulations to analyze composite materials and structures at the meso and macro levels. Specifically, we are deal with composite materials and structures constituted by an elastic matrix reinforced by long or short elastoplastic fibers that may debound when the contact stress reaches a critical value. Before starting the description of our approach it is important to mention reference [1], in which one finds an important review of the subject and a specific formulation (different from ours) to model the behavior of fiber reinforced composite and structures. Their formulation considers elastic fibers embedded in a physical non-linear matrix considering debounding,small strains and small deformations. Summarizing some points raised by [1] one can say that in FEM literature different ways are developed to incorporate fibers inside matrix domain. The reader is invited to consult the works [1–6] where field enrichments are imposed inside the 2D domain in order to model the fiber–matrix coupling. These enrichments

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are based on general known behavior of fiber–matrix connections. They are mostly based on the so called Partition of Unity FEM [7–12] leading to very elegant and well posed strategies; however the pre-known enrichment field is of difficult achievement when, for example, curved fibers are present. Readers are invited to consult the works [13–15] in which authors employ lattice strategy to model composites from micro-structural consideration. Moreover, other approaches that adopt slip degrees of freedom to represent the fiber reinforced body can be found in [16,17]. Works that employ the Boundary Element Method may also be referred [18,19].

As mentioned above, in this paper we intend to collaborate in meso and macro-level analysis of fiber reinforced solids modeled by Finite Elements. We propose a way to introduce fibers (short or long) inside elastic solids modeled by finite elements without increasing the number of degrees of freedom when debounding is not present. When there is debounding, new degrees of freedom are incorporated only at the slipping contact between fibers and matrix following a non linear constitutive relation. The matrix is considered elastic while fibers follow a multi-linear elastoplastic constitutive relation. Moreover large deformations and moderate strains are considered.

In order to model the debounding between fibers and matrix it is necessary to accurately calculate the contact stresses. Therefore, we present the development of straight and curved fiber elements of any order and analyze the accuracy of the proposed contact stress calculations, defining the best strategy to be followed. A comparison with an analytical contact stress solution [2,20,21] is presented in order to validate the chosen strategy.

The 2D solid finite element applied here to discretize the continuum is isoparametric of any order [22,23]. The adopted nodal parameters are positions, not displacements, which is adequate to model curved elements and large deformations due to the natural presence of a numerical chain rule that solve space transformations. The formulation is classified as total Lagrangian and the Saint-Venant–Kirchhoff constitutive law is chosen to model the matrix material behavior [24,25]. Therefore, the Green strain and the Second Piola–Kirchhoff stress are adopted.

Fiber elements are introduced in matrix by means of nodal kinematic relations. This strategy directly ensures the adhesion of fibers nodes to the matrix without increasing the number of degrees of freedom, when debounding is not considered, and without the need of nodal matching [26,27]. The debounding and fiber non-linear relations are introduced allowing the degeneration of fibers inside elastic bodies.

To solve the resulting geometrical nonlinear problem we adopt the Principle of Stationary Total Potential Energy [28]. From this principle we find the nonlinear equilibrium equations and use the Newton–Raphson iterative procedure [29] to solve the nonlinear system. External loads are considered conservative and incrementally applied.

The paper is organized as follows. Section 2 describes the general nonlinear solution process, indicating the important variables that will be developed in subsequent sections. Section 3 describes the elastic procedure used to model the two-dimensional continuum (matrix). Section 4 presents the developed any order elastic and elastoplastic fiber finite element and describes the chain rule applied to generalize the inclusion of fibers into high order 2D solid finite elements without increasing the number of degrees of freedom. Section 5 presents the proposed fiber–matrix contact stresses calculations in elastic analysis and tests these strategies to choose the best one to incorporate debounding. Section 6 shows how slipping and debounding are considered and Section 7 presents numerical examples comparing and analyzing the behavior of the proposed formulation for general applications. Finally, conclusions are presented in Section 8.

## 2. The general solution process

To solve the proposed fiber reinforced mechanical problem we use the principle of stationary total potential energy. The problem is assumed to be isothermal and external loads are considered conservative. Therefore, the total potential energy is written as:

$$\Pi = \int_{V_0^f} \theta(E,\alpha) dV_0^f + \int_{V_0^b} \Psi(\zeta,\beta) dV_0^b + \int_{V_0^m} u_e(\boldsymbol{E}) dV_0^m - \int_{S_0} \boldsymbol{p}\cdot\boldsymbol{y} ds_0 - \boldsymbol{F}\cdot\boldsymbol{Y} \tag{1}$$

where $\theta$ is the Helmholtz free energy potential of elastoplastic internal fibers, $\Psi$ is the Helmholtz free energy potential of bounding region, $u_e$ is the specific strain energy potential of the elastic matrix, $\boldsymbol{F}$ is the concentrated external conservative load vector, $\boldsymbol{Y}$ is the nodal position vector, including all degrees of freedom, $\boldsymbol{p}$ is the conservative distributed load related to the surface position $\boldsymbol{y}$. Moreover, $E$ is the uniaxial Green–Lagrange strain measured at fibers, $\zeta$ is the relative dimensionless position of fiber and matrix understood as the bounding slip, $\boldsymbol{E}$ is the Green–Lagrange strain developed at the elastic matrix, $\alpha$ is the internal variable that controls the plastic strain evolution of fibers and $\beta$ is the internal variable that controls the non-linear slipping between fibers and matrix.

A variation of Eq. (1) is equal to zero at equilibrium position, that is, the problem consists on finding a position $\boldsymbol{Y}$ that satisfies the assumption:

$$\delta\Pi = 0 \tag{2}$$

As the main variable of the problem is $\boldsymbol{Y}$ we rewrite Eq. (2) as,

$$\delta\Pi = \int_{V_0^f} \frac{\partial\theta}{\partial\boldsymbol{Y}} dV_0^f \cdot \delta\boldsymbol{Y} + \int_{V_0^b} \frac{\partial\Psi}{\partial\boldsymbol{Y}} dV_0^b \cdot \delta\boldsymbol{Y} + \int_{V_0^m} \frac{\partial u_e}{\partial\boldsymbol{Y}} dV_0^m \cdot \delta\boldsymbol{Y} - \int_{S_0} \boldsymbol{\phi}\otimes\boldsymbol{\phi} ds_0 \boldsymbol{P}\cdot\delta\boldsymbol{Y} - \boldsymbol{F}\cdot\delta\boldsymbol{Y} \tag{3}$$

In which $\boldsymbol{\phi}$ is the set of shape functions related to the surface of elements and $\boldsymbol{P}$ is the nodal value of distributed applied forces. The first, second and third terms of Eq. (3) are further developed,

$$\frac{\partial\theta}{\partial Y_k} = \frac{\partial\theta}{\partial E}\frac{\partial E}{\partial Y_k}, \quad \frac{\partial\Psi}{\partial Y_k} = \frac{\partial\Psi}{\partial\zeta}\cdot\frac{\partial\zeta}{\partial Y_k} \text{ and } \frac{\partial u_e}{\partial Y_k} = \frac{\partial u_e}{\partial\boldsymbol{E}} : \frac{\partial\boldsymbol{E}}{\partial Y_k} \tag{4}$$

in which index and dyadic notations are used together.

And, by the energy conjugate assumption [25], one writes

$$\frac{\partial\theta}{\partial E} = S \quad \frac{\partial\Psi}{\partial\zeta} = \boldsymbol{q} \quad \frac{\partial u_e}{\partial\boldsymbol{E}} = \boldsymbol{S} \tag{5}$$

in which $S$ is the uniaxial second Piola–Kirchhoff stress developed at the fiber, $\boldsymbol{q}$ is the contact distributed force between fiber and matrix and $\boldsymbol{S}$ is the second Piola–Kirchhoff stress developed in the elastic matrix. The internal variables related to the physical non-linear constitutive relation for fibers and Bounding are omitted in Eqs. (3)–(5) due to their intrinsic relation with $E$ and $\zeta$ to be shown in Sections 4 and 6, respectively.

Introducing Eq. (5) into (3) results:

$$\delta\Pi = \left( \int_{V_0^f} S\frac{\partial E}{\partial\boldsymbol{Y}} dV_0^f + \int_{V_0^b} \boldsymbol{q}\cdot\frac{\partial\zeta}{\partial\boldsymbol{Y}} dV_0^b + \int_{V_0^m} \boldsymbol{S} : \frac{\partial\boldsymbol{E}_e}{\partial\boldsymbol{Y}} dV_0^m - \int_{S_0} \boldsymbol{\phi}\otimes\boldsymbol{\phi} ds_0 \boldsymbol{P} - \boldsymbol{F} \right)\cdot\delta\boldsymbol{Y} = \boldsymbol{0} \tag{6}$$

This equation is clearly nonlinear and the Newton–Raphson solution process starts rewriting the equilibrium Eq. (6) by removing the arbitrary variation of positions $\delta\boldsymbol{Y}$ and defining the unbalanced force vector $\boldsymbol{g}$, as:

$$g(\boldsymbol{Y}) = \left( \int_{V_0^f} S\frac{\partial E}{\partial\boldsymbol{Y}} dV_0^f + \int_{V_0^b} \boldsymbol{q}\cdot\frac{\partial\zeta}{\partial\boldsymbol{Y}} dV_0^b + \int_{V_0^m} \boldsymbol{S} : \frac{\partial\boldsymbol{E}}{\partial\boldsymbol{Y}} dV_0^m \right) - \boldsymbol{M}\cdot\boldsymbol{P} - \boldsymbol{F} = \boldsymbol{0} \tag{7}$$