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Flutter analysis of fixed and rotary wings through a one-dimensional unified formulation

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ABSTRACT

Carrera Unified Formulation (CUF) is used to perform flutter analyses of fixed and rotary wings. The one-dimensional refined theories are obtained through an axiomatic enrichment of the displacement field components by only setting the input parameters, namely the number of terms and the kind of the cross-sectional functions. Within this work, Taylor-like expansions of *N*-order (TE*N*) are used. The aerodynamic loadings are determined through the unsteady strip theories proposed by Theodorsen and Loewy. The finite element method is used to solve the governing equations that are derived, in a weak form, using the generalized Hamilton's Principle. These equations are written in terms of CUF "fundamental nuclei", which do not vary with the theory order (*N*). The flutter instability of fixed and rotary wings with rectangular and realistic cross-sections is investigated. The results are reported in terms of flutter velocities and frequencies and, when possible, they are compared with experimental, numerical and analytical solutions. Despite the intrinsic limitations of the used aerodynamic theories, the proposed methodology appears valid for aeroelastic and vibrational analyses of several structures by ensuring a significant accuracy with a low computational cost.

1. Introduction

According to Collar's definition, aeroelasticity is "the study of the mutual interaction that takes place within the triangle of the inertial, elastic, and aerodynamic forces acting on structural members exposed to an air stream, and the influence of this study on design". The fluid–structure interaction (FSI) has brought to catastrophic events due to sudden failures of bridges, airplanes, helicopter blades, etc. A correct and safe design must require, therefore, an accurate prediction of aeroelastic phenomena. Unfortunately, FSI analyses are, in most cases, too computationally expensive; thus a tradeoff between accuracy and cost is often necessary.

For fixed-wing configurations, the simplest aerodynamic theories based on strip approaches were proposed in the first half of the twentieth century [1–5]. These models were properly combined with simplified structural theories to develop reliable aeroelastic tools. For instance, aeroelastic analyses were performed on wings [6–8] and civil structures [9] by using the beam-plate approach. On the basis of this methodology, the flexural, torsional and secondary stiffness of the cross-section are reduced to the equivalent beam quantities (EI, GJ, K, etc.). To overcome the intrinsic limitations of this technique (no coupling due to the material distribution), Librescu et al. proposed a refined 1D theory including non-classical effects, such as warping and transverse shear deformations and the non-uniform torsion [10–12]. Moreover, Patil et al. developed a non-linear 1D formulation for laminated box beams by adopting an asymptotic technique to compute the cross-sectional mechanical properties [13,14]. Later, Palacios et al. used the same formulation to evaluate the aeroelastic behavior of highly flexible wings [15]. Nowadays, the rapid increase of the computational power has motivated the development in the area of computational fluid dynamics (CFD). Over the last 30 years, the CFD-based aeroelasticity progressed from full potential theories (strip theory, panel methods) to problems governed by the Navier–Stokes equations. Recent works offer interesting overviews of CFD-aeroelastic tools [16,17] furthermore providing useful information about the emerging trends in the aeroelasticity field [18]. From a structural point of view, the CFD simulations can be coupled with finite element models, in which non-linear elements may be used to include either geometrical effects or large deformations. A detailed paper about non-linear structural models can be found in [19].

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Regarding the aeroelasticity of rotary wings, the problem becomes more complex due to the geometric nonlinearities that must be taken into account in both elastic and aerodynamic terms. This need has driven the development of suitable structural and aerodynamic theories as confirmed by the considerable number of articles available in the literature. Almost 50 years ago, Loewy provided a thorough overview on a broad range of topics related to the dynamics and aeroelasticity of rotary wings such as flap-lag flutter, pitch-lag flutter and ground and air resonance [20]. Although being more limited in scope, other contributions to the study of classical flutter conditions and unsteady aerodynamic theories were proposed in [21,22], respectively. In that period, the aerodynamic models for the instability predictions were essentially extensions of steady and quasi-steady strip theories conceived for fixed-wing configurations. The first important unsteady approach for hover, based on Theodorsen's model, was proposed by Loewy [23]. Though in an approximate manner, this theory takes into account the effect of the spiral returning wake beneath the rotor. Other valuable models were proposed on the basis of Greenberg's theory [24], where a pulsating velocity variation and a constant pitch angle were included in Theodorsen's model. These modifications were extended to the case of rotary wings in [25], in which Theodorsen's, Loewy's and Poisso's theories were used for studying the flap-lag-torsional coupling. The authors pointed out that the above modifications should be included to realistically reproduce the aeroelasticity of rotary wings, where the assumptions commonly used in deriving strip theories (1 – cross sections are assumed to perform only simple harmonic pitching and plunging oscillations about a zero equilibrium position; 2 – the velocity of oncoming airflow is constant; 3 – usual potential small-disturbance unsteady aerodynamics are assumed to apply) are intrinsically violated. The modified strip theory with the quasi-steady approximation was adopted to study the stability of composite helicopter blades with swept tips both in hover and in forward flight [26]. The swept-tip blades were modeled using non-linear 1D finite element with the inclusion of transverse shear deformations and out-of-plane warping. These results were recently used to verify the accuracy of a nonlinear structural formulation valid for slender, homogeneous and twisted blades [27]. Other analyses on advanced geometry blades were carried out in [28] with the purpose of evaluating the effects of sweep and droop on both rotor aeroelastic stability and rotorcraft aero-mechanical stability. The evolution of the mechanics of helicopter blades, focusing on the aeroelastic and aerodynamic issues, was extensively discussed in [29–32]. Moreover, even if the aeroelasticity of large wind turbine blades is inherently different, several related aeroelastic problems were solved using rotary wing theories [33–35]. Furthermore, the response problems of an isolated wind turbine blade and a complete rotor/tower configuration were detailed discussed in [36–38].

Within this work, we propose an advanced 1D formulation to predict the flutter conditions of various fixed and rotary wing configurations. The higher-order beam theories are obtained using the Carrera Unified Formulation (CUF), which enables to derive, at least theoretically, an infinite number of models [39–41]. The Taylor-type expansions (referred to as TE) represent a particular class of the 1D-CUF theories. The TE models have been adopted for the study of mechanical behaviors of thin-walled structures [42,43] and non-conventional cross-sections made of isotropic, composite [39,40] and functionally graded [41] materials. Other encouraging results have also been obtained in the dynamics of rotors by analyzing both blades [44] and spinning shafts [45–47]. Furthermore, aeroelastic analyzes were performed on fixed-wing configurations by combining the TE elements with aerodynamic panel methods [48,49] and unsteady strip theories [50].

2. The structural model: Carrera Unified Formulation

CUF states that the displacement field, $\boldsymbol{u}^T(x,y,z,t) = (u_x, u_y, u_z)^T$, is an expansion of generic functions, $F_\tau(x,z)$ for the displacement vector, $\boldsymbol{u}_\tau(y)$:

$$\boldsymbol{u}(x,y,z,t) = F_\tau(x,z)\boldsymbol{u}_\tau(y,t) \quad \tau = 1,2,\ldots,T \tag{1}$$

where T is the number of terms in the expansion and, according to Einstein's generalized notation, τ indicates summation. In this work, Eq. (1) consists of Taylor-like expansions denoted as '*TE*'. For example, the third-order displacement field (TE3) is:

$$\begin{aligned}
u_x &= u_{x_1} + x\, u_{x_2} + z\, u_{x_3} + x^2\, u_{x_4} + xz\, u_{x_5} + z^2\, u_{x_6} + x^3\, u_{x_7} \\
&\quad + x^2 z\, u_{x_8} + xz^2\, u_{x_9} + z^3\, u_{x_{10}} \\
u_y &= u_{y_1} + x\, u_{y_2} + z\, u_{y_3} + x^2\, u_{y_4} + xz\, u_{y_5} + z^2\, u_{y_6} + x^3\, u_{y_7} + x^2 z\, u_{y_8} \\
&\quad + xz^2\, u_{y_9} + z^3\, u_{y_{10}} \\
u_z &= u_{z_1} + x\, u_{z_2} + z\, u_{z_3} + x^2\, u_{z_4} + xz\, u_{z_5} + z^2\, u_{z_6} + x^3\, u_{z_7} \\
&\quad + x^2 z\, u_{z_8} + xz^2\, u_{z_9} + z^3\, u_{z_{10}}
\end{aligned}$$

The remarkable feature of these models is that classical beam theories can be obtained as particular cases of Taylor expansions. Although, the capabilities of the TE approaches have been evaluated in several works, we suggest referring to [51] for a detailed description. The stresses and the strains are grouped as follows:

$$\begin{aligned}
\boldsymbol{\epsilon}_p &= \{\epsilon_{zz} \quad \epsilon_{xx} \quad \epsilon_{xz}\}^T \quad \boldsymbol{\sigma}_p = \{\sigma_{zz} \quad \sigma_{xx} \quad \sigma_{xz}\}^T \\
\boldsymbol{\epsilon}_n &= \{\epsilon_{zy} \quad \epsilon_{xy} \quad \epsilon_{yy}\}^T \quad \boldsymbol{\sigma}_n = \{\sigma_{zy} \quad \sigma_{xy} \quad \sigma_{yy}\}^T
\end{aligned} \tag{2}$$

where the subscript "p" stands for the terms lying on the cross-section, while "n" stands for the terms lying on the other planes, which are orthogonal to the cross-section. The strain–displacement relations and Hooke's law are, respectively:

$$\begin{aligned}
\boldsymbol{\epsilon}_p &= \boldsymbol{D}_p\mathbf{u} \\
\boldsymbol{\epsilon}_n &= \boldsymbol{D}_n\mathbf{u} = (\boldsymbol{D}_{np} + \boldsymbol{D}_{ny})\mathbf{u}
\end{aligned} \tag{3}$$

$$\begin{aligned}
\boldsymbol{\sigma}_p &= \tilde{\boldsymbol{C}}_{pp}\boldsymbol{\epsilon}_p + \tilde{\boldsymbol{C}}_{pn}\boldsymbol{\epsilon}_n \\
\boldsymbol{\sigma}_n &= \tilde{\boldsymbol{C}}_{np}\boldsymbol{\epsilon}_p + \tilde{\boldsymbol{C}}_{nn}\boldsymbol{\epsilon}_n
\end{aligned} \tag{4}$$

where $\boldsymbol{D}_p$ and $\boldsymbol{D}_n$ are linear differential operators. Both laminated box beams and cylinders can be considered constituted by a certain number of either straight or curved plates of orthotropic material, whose material coordinate systems generally do not coincide with the physical coordinate system (x,y,z) of the structure. The matrices of material coefficients of the generic material k are

$$\begin{aligned}
\tilde{\boldsymbol{C}}^k_{pp} &= \begin{bmatrix} \tilde{C}^k_{11} & \tilde{C}^k_{12} & \tilde{C}^k_{14} \\ \tilde{C}^k_{12} & \tilde{C}^k_{22} & \tilde{C}^k_{24} \\ \tilde{C}^k_{14} & \tilde{C}^k_{24} & \tilde{C}^k_{44} \end{bmatrix}, \qquad
\tilde{\boldsymbol{C}}^k_{pn} = \begin{bmatrix} \tilde{C}^k_{15} & \tilde{C}^k_{16} & \tilde{C}^k_{13} \\ \tilde{C}^k_{25} & \tilde{C}^k_{26} & \tilde{C}^k_{23} \\ \tilde{C}^k_{45} & \tilde{C}^k_{46} & \tilde{C}^k_{43} \end{bmatrix}, \\
\tilde{\boldsymbol{C}}^k_{nn} &= \begin{bmatrix} \tilde{C}^k_{55} & \tilde{C}^k_{56} & \tilde{C}^k_{35} \\ \tilde{C}^k_{56} & \tilde{C}^k_{66} & \tilde{C}^k_{36} \\ \tilde{C}^k_{35} & \tilde{C}^k_{36} & \tilde{C}^k_{33} \end{bmatrix}
\end{aligned} \tag{5}$$

For the sake of the brevity, the explicit forms of the coefficients of the $\tilde{\boldsymbol{C}}$ matrices are shown in [46]. The finite element method is used in order to consider arbitrary cross-section profiles. The generalized displacement vector is

$$\mathbf{u}_\tau(y,t) = N_i(y)\mathbf{q}_{\tau i}(t) \tag{6}$$

where $N_i(y)$ are the shape functions and $\mathbf{q}_{\tau i}(t)$ is the nodal displacement vector

$$\mathbf{q}_{\tau i}(t) = \left\{ q_{u_{x_{\tau i}}} \quad q_{u_{y_{\tau i}}} \quad q_{u_{z_{\tau i}}} \right\}^T \tag{7}$$