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Various approaches in probabilistic homogenization of the CFRP composites

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ABSTRACT

A variety of the homogenization methods is collected here and contrasted to determine the basic probabilistic characteristics of the CFRP composite in a presence of Gaussian uncertainty in its components material characteristics. We compare algebraic approximations of the effective elasticity tensor components with these calculated by using of two different FEM strategies. One may find in this elaboration a comparison of the Voigt–Reuss versus Hashin–Shtrikmann upper and lower bounds, the results of spatial averaging of this composite RVE, Mori–Tanaka, Vanishing Fiber Diameter as well as the Self-Consistent homogenization theories together with two independent FEM solutions of the so-called cell problem. Symbolic probabilistic analysis is implemented according to the generalized tenth order stochastic perturbation technique supported by the Response Function Method. Polynomial basis with statistically optimized order enables determination of the analytical functions relating effective tensor components with material parameters of the composite constituents. The expected values series confirm fundamental tendencies observed before in deterministic comparisons of these theories, an analysis of the coefficients of variations shows the resulting uncertainty level and sensitivity of the effective characteristics, while skewness and kurtosis show that Gaussian character of the homogenized tensor prevails in this problem.
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1. Introduction

Various homogenization methods are well known and compared against each other at various occasions since many years. This was done in different due to different constituents combinations, various types of the reinforcements, matrices or fillers as well as their various shapes and distributions in the Representative Volume Element (RVE) [8,18]; some numerical studies contrasted algebraic approximations of the effective tensor contrary to the these related to the Finite Element Method (FEM) [19] solution of the so-called cell problem. The main goal of this work is to collect most of the well-known homogenization approaches in the view of their common randomization and a contrast of the first four statistics – expectations, coefficients of variation, skewness and kurtosis – to check whether they exhibit some similarities in the input uncertainty preservation or amplification. All material characteristics of the Carbon Fiber Reinforced Polymers (CFRP) [3,4,7,15,17] chosen in this study are separately randomized according to the Gaussian distribution [10], so that the overall output distributions of the effective tensor components are also examined to verify whether they can have the same distribution while homogenizing this composite according to all the mathematical theories contrasted here. One may find in this study the mixture rules (spatial averaging procedure) [8], upper and lower bounds on the effective tensor proposed by Voigt and Reuss as well as by Hashin and Shtrikman [1,12], some algebraic equations for such a tensor available as Vanishing Fiber Diameter (VFD) method [2], the Self-Consistent (SC) method [5,6], Mori–Tanaka (MT) approximates [16]. Additionally, two FEM-based approaches are randomized also and this is the energy method (uniform extension of the RVE with constant deformation provided in the system ABAQUS) as well as the stress method (with periodic boundary conditions on the outer edges of the RVE and under specific interface stresses) [8,10]. Having all these methods included into a single complex probabilistic procedure we compare and discuss their assumptions, input parameters as well as the homogenized medium properties. Similar computational study restricted to the first two probabilistic characteristics has been provisionally presented in [14].

Probabilistic methods available today include quite different theoretical and numerical techniques [10], where one may deal

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with (a) exact probabilities of some effective tensor values (Bayesian approach), (b) statistical estimators family for this tensor (Monte-Carlo simulation), (c) analytical or semi-analytical derivations of the probabilistic moments (i.e. via symbolic integration with a priori known algebraic formulas including composite components characteristics), (d) expansion methods (with polynomial chaos or Karhunen–Loeve methods), (e) stochastic perturbation method (in the second or in higher order formulations relevant to the Taylor series representation of random variables). Application of the fuzzy sets theory is also possible but its application in the homogenization-based analysis of composites remains unavailable right now. Higher order statistics determination is fully available, together with the additional probabilistic analysis, for the Monte-Carlo scheme and higher order stochastic perturbation technique. However, statistical method needs a tremendous number of random samples (cell problem solutions) in its crude implementation to obtain a satisfactory accuracy of both skewness and kurtosis, while stochastic perturbation technique computational cost is close to a deterministic homogenization problem solution with very similar accuracy. Some other majority of the perturbation approach behind the simulation technique is in the fact that the uncertainty level as a separate parameter is inherent into computational procedure in the first method only (including automatic determination of the sensitivity coefficients as first order partial derivatives are calculated by the way), and Monte-Carlo always returns some numerical values in the single numerical experiment. The previous study concerning similar subject [11] that the perturbation technique has the same accuracy in homogenization of the CFRP as the Monte-Carlo scheme and, separately, the semi-analytical strategy, also implemented on the basis of the LSM polynomial fitting but with non-optimized basis. Therefore, a common randomization of all the aforementioned homogenization methods is carried out in this study via the generalized stochastic perturbation technique by using of the polynomial basis of statistically optimized order not larger than the tenth. The Response Function Method (RFM) constructed upon the Least Squares Method (LSM) is applied to numerically recover all polynomial interrelations of the effective tensor components with respect to the original material characteristics of the CFRP constituents. An optimization of such polynomials orders is carried out by minimization of the variance of the LSM fitting and maximization of the correlation of the approximating polynomial to the set of trial values obtained during several series of symbolic computations; probabilistic part and the LSM self-optimizing procedure are entirely realized in the computer algebra system MAPLE®.

A major novelty in this study is in complete and reliable contrastive comparison of the first four probabilistic characteristics of the effective elasticity tensor as the functions of the input randomness in Young moduli and Poisson ratios of the composite constituents with the smallest available computer time and power effort. The very interesting result of such a study is that the hierarchy of expectations in-between various homogenization techniques remains the same as for their deterministic counterparts, while higher order probabilistic coefficients show a lot of regularities. One of them is an output uncertainty generally limited to that assumed for input random parameters as well as skewness and kurtosis similar to these adjacent to the Gaussian distribution. This verification is of paramount importance – in the view of such an observation one may conclude that calculation of the first two statistical moments is really needed, while all higher order moments may be recovered by some simple algebra from the recursive formulas available in case of normal distribution. It should be finally mentioned that the general motivation of such a research direction is to implement and to test computational basis for the future full stochastic reliability (and also durability) analysis of the CFRP structures or their reinforcements attached to the reinforced concrete elements including input statistics of the constituents by using of the homogenization technique.

2. CFRP mathematical model

The main objective of our considerations is periodic fiber–matrix composite structure Y, where the cross-section with the plane perpendicular to the fibers longitudinal direction is constant and is given schematically in Fig. 1 in the micro-scale (upper diagram) and also in the micro-scale: the Representative Volume Element (RVE) equivalent to the single periodicity cell is shown in lower graph and denoted here by Ω. It is meant that geometrical diameter of this RVE is related to the corresponding geometrical parameter of the entire composite structure with some small parameter $\varepsilon > 0$. Therefore, it is assumed that we consider the composite with long and parallel fibers that are perfectly bonded with the surrounding statistically homogeneous matrix; both components are linear elastic and isotropic.

All elastic parameters (Young modulus E_f and Poisson ratio ν_f for the fiber and similarly, E_m, ν_m – for the matrix) are further treated statistically – we define them separately as fully uncorrelated Gaussian random variables having specified expected values: $E[E_f]$, $E[\nu_f]$ and $E[E^m]$, $E[\nu_m]$, correspondingly, while their coefficients of variation $\alpha(E_f)$, $\alpha(\nu_f)$ and $\alpha(E_m), \alpha(\nu_m)$ are the additional parameters in the CFRP model. Assumed periodicity of such a composite means that these elastic parameters have the same probability distributions in each RVE, so that periodicity sense is a little bit wider than for the deterministic case. It also means that both components are statistically homogeneous, which admits some local non-homogeneities inside the fiber and in the matrix separately, but does not demand any correlation function. Such a definition reflects perfectly the experimental information

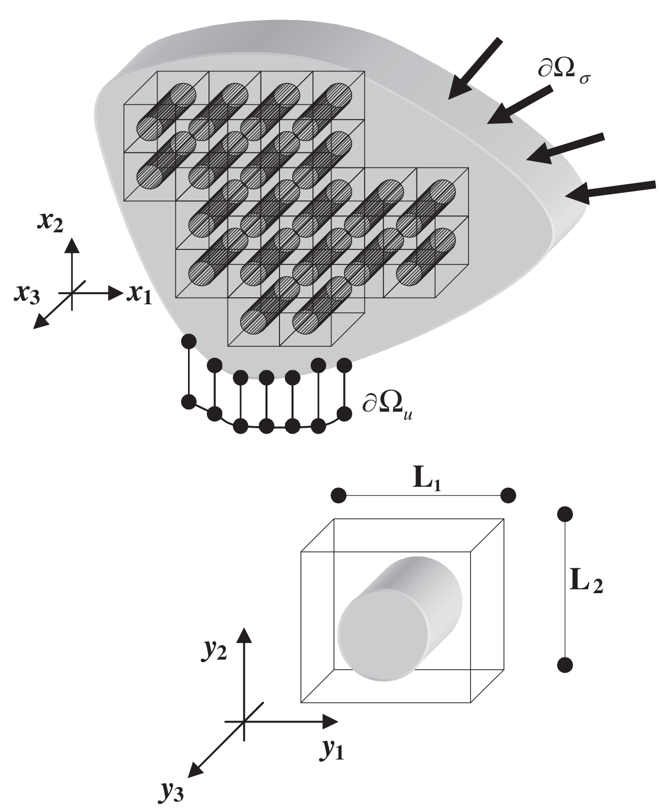

Fig. 1. Periodic composite structure and the RVE.

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