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State space formulation of magneto-electro-elasticity in Hamiltonian system and applications

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ABSTRACT

A systematical method is presented to derive the state space equations of anisotropic magneto-electro-elastic materials of orthogonal curvilinear coordinates in Hamiltonian system. Based on the three-dimensional theory of magneto-electro-elasticity, the constitutive relations and the Lagrangian function of the total potential are rewritten in terms of the generalized displacements, i.e., displacements, electric potential and magnetic potential, and their derivatives. The Legendre transform is then applied to release the Hamiltonian function and the canonical equations are obtained through variational operation. The canonical equations in curvilinear coordinates, i.e., the desired state equations, can be degenerated readily to the ones of spherical coordinates, cylindrical coordinates and rectangular coordinates. These equations are also deduced conveniently for piezoelectric and elastic materials since these derivation are rendered in terms of matrix. As an example of application, a magneto-electro-elastic cylindrical shell with four simply supported edges is studied when it is bended by sinusoidal load.

1. Introduction

The state space method could be traced back to the initial function method by Vlasov in 1950s [1,2], in which three displacement components and three stress components were chosen as state vector functions and combined in one mixed equation set including displacements and stresses. These state vectors were expanded in terms of the Maclaurin series along a specific coordinate and the boundary value problems of the elasticity became the initial problems with respect to the specific coordinate. Some approximate theories of beams and plates of various orders could be obtained systematically by cutting the infinite series of Maclaurin to the desired order. Lur'e [3] also adopted similar symbolic operators to solve three-dimensional problems in the theory of elasticity. The method of symbolic operators were extended for dynamic problems in two-dimension by Das and Setlur [4] and further for orthogonal anisotropic materials by Xu and Ding [5]. Rao and Das [6] used this method to study the dynamic behavior of thick plates of isotropic materials and got the solution of a rectangular plate with four simply supported edges. The Navier solution of the rectangular plate with four simply supported edges and the Levy solution of the rectangular plate with two simply supported opposite edges were given by Iyengar et al. [7]. Although the method of symbolic operator from Vlasov [1,2] and Lur'e [3] showed the versatility in the derivation of approximate theories, they were limited in theories of beams and plates and few applications in the theory of elasticity because it was difficult to identify the differential operator in terms of infinite series to an elementary function in closed form, especially for non-Cartesian coordinates. For this purpose, Bahar [8] introduced the concept of state space in modern control theory and the theorem of Cayley–Hamilton to avoid the differential equation in the form of infinite series and to be suitable for non-Cartesian coordinates. This method was used by Stroh [9] and Ingebrigtsen and Tonning [10] to investigate the wave motions of anisotropy. It was also used by Celep [11] to study the free axisymmetric vibration of isotropic circular plates and that of transversely isotropic circular plates by Fan and Ye [12]. Unfortunately, these solutions involved some fatal errors in misusage of Bessel functions to describe the distribution of the displacements and stresses in the radial direction of the circular plates, which was found by Ding et al. [13]. They also corrected these errors and extended the method to the axisymmetric bending and free vibration of piezoelectric circular plates. The state space method was also used by Fan and Ye [14] to investigate the bending and free vibration of orthogonal laminated rectangular plates based on three-dimensional theory of elasticity. The cases of cylindrical shells and doubly curved shells were also presented by

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Table 1
Values of the parameters [*a*] and [*b*].

α, β	[*a*], [*b*]
ξ	[1,6,5,7,10]
η	[6,2,4,8,11]
ζ	[5,4,3,9,12]

Table 2
Parameters [*a*] and [*b*] for piezoelectricity.

α, β	[*a*], [*b*]
ξ	[1,6,5,7]
η	[6,2,4,8]
ζ	[5,4,3,9]

Table 3
Parameters [*a*] and [*b*] for elasticity.

α, β	[*a*], [*b*]
ξ	[1,6,5]
η	[6,2,4]
ζ	[5,4,3]

Fan and Zhang [15,16] through the same method. In 1990s, electric components made by piezoelectric materials were developed rapidly and attracted more attentions of researchers. The electromechanical coupling effect of piezoelectric materials and structures was the key property and was study by the state space method [17–21]. In the new century, the state space method was applied widely in the plates and shells of functionally graded materials (FGMs) [22–25], on which many research focused at that time. With the gradual development of the state space method, some extensions in different aspects were made. For examples, Ding et al. [26] derived the state space equations of displacement functions and stress functions [26]. Chen and co-worker [27–31] introduced the spring models of imperfect interfaces into the state space method to illustrate the effect of the imperfect interfaces on the laminated plates and shells. The differential quadrature method (DQM) was also combined with the state space method to deal with those non-simply supported edges [32–34] as well as the interaction between the structures and fluid [35]. Recently, Chen [36] established a theoretical framework of surface elasticity based on the state space method and symbolical operator. The above-mentioned researches showed that the state space method became versatile in theory of elasticity and was widely applied in many structures such as rectangular plates, circular plates, cylindrical shells and doubly curved shells of isotropic, anisotropic, functionally graded, piezoelectric and magneto-electro-thermo-elastic materials in dynamic or static states. The derivation of the state equations was the key procedure

Table 4
Properties of magneto-electro-elastic material.

Properties	$BaTiO_3$–$CoFe_2O_4$ composite
Elastic (GPa)	$c_{11} = c_{22} = 274$, $c_{12} = 163$, $c_{13} = c_{23} = 161$, $c_{33} = 259$ $c_{44} = c_{55} = 45$, $c_{66} = (c_{11} - c_{12})/2$
Dielectric (10^{-10} C^2/Nm^2)	$\varepsilon_{11} = \varepsilon_{22} = 11.9$, $\varepsilon_{33} = 13.4$
Magnetic (10^{-6} Ns^2/C^2)	$\mu_{11} = \mu_{22} = 531.5$, $\mu_{33} = 142.3$
Piezo-electric (C/m^2)	$e_{15} = e_{24} = 1.16$, $e_{31} = e_{32} = -0.44$, $e_{33} = 1.86$
Piezo-magnetic (N/Am)	$d_{15} = d_{24} = 495.0$, $d_{31} = d_{32} = 522.3$, $d_{33} = 629.7$
Magneto-electric	$g_{11} = g_{22} = g_{33} = 0$

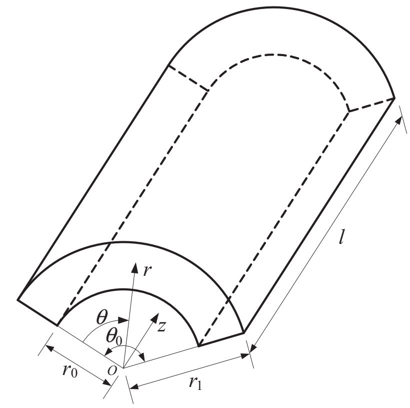

Fig. 1. Geometry of the circular cylindrical panel and the coordinates system.

in the state space method. The state equations were usually obtained from the equations of equilibrium and constitutive relations through the appropriate choices of the state variables in the specific coordinates for the specific problems. Some tedious elimination was also necessary in the derivation. Unlike the traditional methods in the theory of elasticity involving displacements or stresses only, the state space method included both displacements and stresses and it was of great advantage in many problems, especially in laminated structures. However, it remained the idea of the traditional semi-inverse method to solve specific problems in the theory of elasticity.

At the same time of the development of the state space method, Zhong [37] presented a new systematical method to solve the problems of elasticity, which was named as symplectic elasticity. This method was inspired by the dual Hamiltonian system in the classical mechanics. The Lagrangian function of the principle of total potential of elasticity was transformed by Legendre transform into Hamiltonian function and the dualities of the displacements were then obtained rationally as well as the corresponding canonical equations, which was the desired state equations in Hamiltonian system. These earlier researches were mostly published in Chinese and then they were reviewed and introduced into the international world by Lim and Xu [38]. Steel and Kim [39] also obtained the state equations of revolving shells based on the modified variational principle of Hellinger–Reissner. Unlike the original state space method, the state equations in Hamiltonian system could be obtained systematically from the canonical equation of Hamiltonian system as well as the dualities of the displacements,

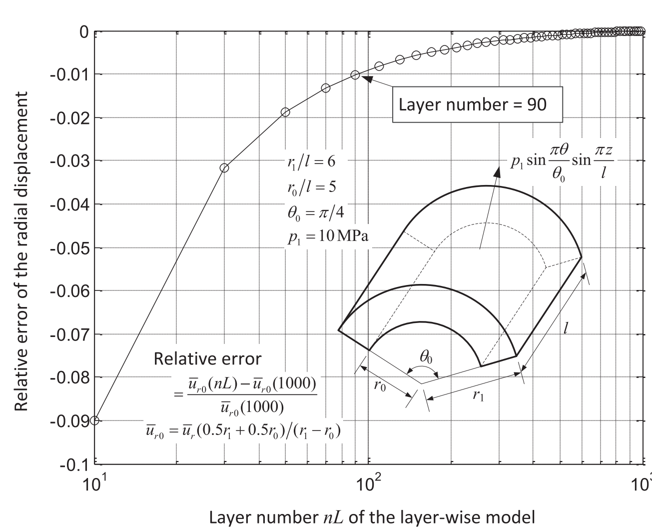

Fig. 2. Convergency of the layer-wise model.