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Numerical analysis of effective mechanical properties of rubber-cord composites under finite strains

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ABSTRACT

In this article, a method and an algorithm are developed by which the effective mechanical properties of rubber-cord composites are numerically estimated, with the finite strains and low compressibility of rubber accounted for. The effective properties are derived in the form of a quadratic relation between the Green deformation tensor and the second Piola–Kirchhoff stress tensor. Numerical results obtained using implementation of this algorithm show good agreement with analytical formulas. The results of the numerical estimation of the effective mechanical properties of rubber-cord composite under finite strains are given in the article. The numerical analysis is carried out using the finite element method. Dependences of the effective modules on the elastic properties of cord and rubber and on the cord pitch are investigated for single-layer rubber-cord. Dependences of the effective modules on the cord angle are investigated for two-layer rubber-cord. Graphs of these dependences are given in the article.

1. Introduction

Rubber-cord composite is rubberized layers of cord fibers. Used primarily in the tire industry, rubber cord is used to make the carcass and breaker in the production of pneumatic.

In the numerical strength analysis of a tire's stress–strain state as a solid construction, it is important to compute the maximum stresses in rubber between cord fibers with sufficiently high accuracy. The lifetime of a tire depends on these stresses, and this dependence is substantially nonlinear: a 5% change in maximum stress can cause a one-and-a-half change in resource. The modeling of each cord fiber is necessary for the exact calculation of maximum stresses in rubber; however, it is only relevant in a region of maximum stress, which is usually the tire's contact patch with the road.

It is not necessary to model the rubber and cord separately in the other parts of a tire. Moreover, the modeling of each cord fiber is a waste of computing resources, because the typical tire size is about a meter and the typical cord-fiber diameter is about a millimeter. Thus, it is reasonable to model the rubber-cord composite outside of the areas of stress concentration on the rubber by a solid material in which mechanical behavior corresponds, on average, to the initial composite. This material is called effective material and its mechanical properties are called effective properties. In this study, we pose a question: How do we estimate the numerically effective mechanical properties of rubber cord if the geometry and mechanical properties of the rubber and cord and known?.

Averaging the mechanical properties of inhomogeneous materials has interested scientists since the middle of the last century. The concept of the representative volume element (RVE) and theoretical principles of such averaging are described in [10]. The effective properties of composites have been studied in previous articles in a linear form, which is suitable for modeling the behavior of composites under small strains.

Hashin–Shtrikman bounds [8,9] are valid for two-phase composites with a relatively small volume content of filler in the matrix. These bounds give the minimum and maximum values for a composite's bulk modulus and shear modulus by a known concentration of the filler, the filler modules, and the matrix. The Mori–Tanaka method [18] is the application of Hashin–Shtrikman bounds to a well-ordered particulate composite with isotropic spherical solid inclusions in a discontinuous matrix. Analytical formulas for the estimation of effective elastic properties of a particulate fiber and layered composites are given in [5]. In that paper, plastic and viscoelastic effects and effective thermal properties of composites are also considered.

Abbreviations: RVE, representative volume element; FEM, finite element method; BC, boundary condition; CAE, computer-aided engineering.

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Currently, the more important problem is the estimation of effective mechanical properties for inhomogeneous materials in a nonlinear form, which is suitable for modeling composite behavior under finite strains. Elastic and plastic properties of a material containing distributed microvoids with different orientations are studied in [4]. Effective elastic properties of solids with cavities of various shapes and orientations are estimated in [25,26]. The finite element method (FEM) is used in [22] for deriving effective viscoelastic constitutive equations; it is assumed that a material has periodic structure. The 2D problem of theory of elasticity for RVE is solved using FEM and the results are then averaged. The application of a variational principle to estimate the effective properties of a multicomponent composite in the form of strain-energy density is described in [19,20]. An estimation method of the composite effective properties in a nonlinear form on the basis of the Hashin–Shtrikman [8,9] principle is given in [23]. A method of deriving nonlinear thermoviscoelastic effective properties for periodical composites with a rubber-like matrix is described in [1]. The application of probability-theory methods for the estimation of effective mechanical properties of irregular composites is given in [11]. A method suitable for the estimation both of elastic and conductive properties of particulate materials is given in [12]. In this article, the authors investigate the influence of different parameters of reinforcing particles on effective elastic and conductive properties. The averaging of elastic properties of inhomogeneous material in [7] is carried out by nonperiodical boundary conditions and taking geometric nonlinearity into account. Practical implementation is carried out using FEM (in the case of 2D). This approach is extended to a multiscale case in [6]. Some methods of homogenization of linear viscoelastic and nonlinear viscoplastic composites are compared in [17]. Effective properties and microcracking in textile composites are represented in [24] using FEM. Some effective modules (bending stiffness and lateral compression stiffness) of rubber-cord layer are derived in [21].

Calculating the rubber-cord effective properties is a problem that involves some peculiarities. The rubber-cord is an anisotropic reinforced material involving rubber, a weakly compressible and highly elastic material. Modules of elasticity of rubber and cord can vary by 3–4 orders of magnitude [2]. These rubber-cord properties can cause difficulties in analyzing its stress–strain state, which is necessary for the estimation of effective properties.

In this article, we propose an approach to deriving effective properties of composites under finite strains. The basic principles underlying this approach are the following [3,13–15,29]. The representative volume element (RVE) in the form of a rectangular parallelepiped is allocated in the undeformed-configuration composite. An effective material is a homogeneous material that satisfies the following condition. If we fill the RVE with this homogeneous material and the same RVE with source composite, then the averaged stresses over the volume in source composite and the homogeneous effective material will be equal for the equal displacements of bounds. Constitutive equations for an effective material are derived in the form of a quadratic relation between the Green deformation tensor and the second Piola–Kirchhoff stress tensor. For calculating the coefficients of this relation we solve several sets of boundary value problems for the RVE with assigned boundary displacements. The method of boundary-condition formation for these task sets is proposed in this article, which takes the low compressibility of rubber into account.

The results of the analysis of the effective characteristics of rubber-cord material are derived using the developed program module. The mechanical properties of cord in the calculation are modeled using the Murnaghan potential [16]. The mechanical properties of rubber are modeled using the Mooney–Rivlin potential [16] modified in order to take low compressibility of rubber into account. The dependences of the effective modules of the single-layer rubber-cord on the elastic properties of cord and rubber and on the cord pitch are studied. The dependences of the effective modules on the cord angle are studied for two-layer rubber-cord.

2. Algorithm for analysis of the effective mechanical properties of rubber-cord composite

Let us consider the basic notations that will be used in the description of the algorithm for the analysis of the effective properties of rubber cord.

$\overset{0}{R}, R$	the radius-vector of a particle in the initial and current states;
$u = R - \overset{0}{R}$	the displacement vector;
ξ^i	the material coordinates of a particle;
x^i	the spatial coordinates of a particle;
e^i	the basic vectors of coordinate system;
$\underset{0}{\ni}_i = \frac{\partial \overset{0}{R}}{\partial \xi^i},\ \ni_i = \frac{\partial R}{\partial \xi^i}$	the basic vectors in the initial and current states;
$\overset{0}{\nabla} = \overset{0}{\underset{i}{\ni}} \frac{\partial}{\partial \xi^i},\ \nabla = \ni_i \frac{\partial}{\partial \xi^i}$	the gradient operators in the initial and current states;
I	the identity tensor;
$F = \left(\overset{0}{\nabla} R\right) = \left(I + \overset{0}{\nabla} u\right) = \left(\nabla \overset{0}{R}\right)^{-1} = (I - \nabla u)^{-1}$	the deformation gradient;
$\overset{0}{E} = \frac{1}{2}(F^* \cdot F - I) = \frac{1}{2}(\overset{0}{\nabla} u + u \overset{0}{\nabla} + \overset{0}{\nabla} u \cdot u \overset{0}{\nabla})$	the Green strain tensor;
$E = \frac{1}{2}(I - F^{*-1} \cdot F^{-1}) = \frac{1}{2}(\nabla u + u\nabla - \nabla u \cdot u \nabla)$	the Almansi strain tensor;
$G = F^* \cdot F$	the Cauchy-Green deformation tensor;
$*$	the symbol of transposition;
$:$	the symbol of double scalar convolution;
σ	the true stress tensor;
$P = (\det F) F^{-1} \cdot \sigma$	the first Piola–Kirchhoff stress tensor;
$\overset{0}{S} = (\det F) F^{-1} \cdot \sigma \cdot F^{*-1}$	the second Piola–Kirchhoff stress tensor;
V_0, V	the volume of the RVE in the initial and current states;
Γ_0, Γ	bound of the RVE in the initial and current states;
$\overset{0}{N}, N$	bound normal in the initial and current states.

As stated above, an effective material is a homogeneous material that satisfies the following conditions. If we fill the RVE with this homogeneous material and fill the same RVE with source rubber-cord composite, then the averaged stresses over the volume in source composite and the homogeneous effective material will be equal for equal displacements of bounds. The effective properties are the mechanical properties of this material [13,14].

We will describe the algorithm of the estimation of the effective characteristics of the rubber-cord material, using the above notations.