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# A new procedure for thermo-viscoelastic modelling of composites with general orthotropy and geometry

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ABSTRACT

In this paper, a solution procedure is presented for thermo-viscoelastic analysis of composites in a general form which can be used for general orthotropic materials with simple flat to complicated geometries with thickness change and curvature. Small deformation problems in which the prediction of thermal stresses and/or deformations is of interest, are considered. The incremental solution already available in FE packages is extended to the orthotropic case using available formulations in the literature, making it more general in the sense of having different relaxations in each orthogonal direction. Only the viscoelastic response of the polymeric part of the material should be obtained as input and characterisation tests on the composite material are not needed. The linear orthotropic viscoelastic response of the composite layer is obtained using self-consistent micromechanics equations in the Laplace domain. The material model is implemented in the ANSYS subroutine to define the time-dependent thermo-viscoelastic response of the composite layer. Numerical results are presented which verify the accuracy and applicability of the modelling procedure. The proposed approach can be used later for the residual stress analysis of anisotropic materials including full composites and also complex fuselage panels made of hybrid Fibre Metal Laminates (FMLs).



## 1. Introduction

Polymers exhibit viscoelastic behaviour in the sense of time-dependency of their properties [1,2]. Composite structures with polymeric resins may be used for high-temperature applications in which creep or stress relaxation is present. In order to predict the performance of composites, time–temperature response of the anisotropic material needs to be determined within a thermo-viscoelastic analysis. The viscoelastic property comes from the isotropic resin but when used in the composite material, the thermo-viscoelastic response becomes anisotropic. Different approaches exist in the literature for this purpose that have their own methodologies and of course limitations. In 1997, Zocher et al. [3] formulated the thermo-viscoelastic equations for recursive finite elements programming. They assumed that the anisotropic response of the composite is already known and implemented the recursive equations to integrate the viscoelastic equations for the anisotropic material with a self-generated finite element code. Similar formulation is used by Lee in 2000 [4] to predict the cure induced residual stresses in a AS4/3501-6 composite from the cure dependent viscoelastic data obtained for the material.

Some researchers have measured the viscoelastic response of the composite laminate in order to obtain the orthotropic time dependent stiffness of the material. For example, White and Kim in 1998 [5] have extracted the epoxy dominated effective stiffnesses, i.e. $E_2(t)$ and $G_{12}(t)$, directly from stress relaxation tests performed on AS4-3501-6 prepreg. Since there is no stress relaxation in the fibre direction, other stiffness components were considered as elastic (time-independent). They have also assumed the composite as transversely isotropic and the Poisson's ratio of the epoxy to be constant with temperature and cure. Their finite element code was limited to plane strain problems like long cylinders. Thermo-mechanical nonlinear viscoelastic analysis of composites was performed by Sawant & Muliana in 2008 [6] who presented a numerical scheme for time-stress dependent modelling of epoxies. In their work, a unit cell micromechanical model is incorporated to obtain the orthotropic viscoelastic properties of the composite laminate.

A new procedure is presented in this paper using the elastic or thermo-viscoelastic data of the resin and/or fibre materials. The linear orthotropic viscoelastic response of the composite material

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is obtained and used as the input to the material model programmed in the general finite element package ANSYS. The analysis can be much more general in both anisotropy of the material and also the geometry of the composite structure. The material can be transversely isotropic or orthotropic. Any geometry of the structural component made of the composite material can be modelled in ANSYS pre-processor with any suitable kind of elements. Therefore, problems dealing with beam, plane stress, plane strain and shell type laminates together with 3D solid and solid-shell type composite structures with any complicated geometry or curvature can be modelled. As another example, problems including adhesive bonding of reinforcements, repair patches in composite materials and also splices and doublers in hybrid type ones like Fibre Metal Laminates (FMLs) with flat or curved geometries with even change of thickness are possible to model and analyse in future stages of this study.

In the next phase of this research, orthotropic thermo-viscoelastic properties of the prepreg layers in GLARE panels will be obtained using the viscoelastic data from the epoxy and elastic properties from the glass fibres. The elastic to viscoelastic correspondence principle (Section 2.3) can be used together with the micromechanics equations in the Laplace domain to get the viscoelastic properties for each orthotropic layer. The solution procedure presented here is general for any multi-layered orthotropic material. It is also needed to mention that the numerical analysis can be made using the viscoelastic data of the resin with thermorheologically simple or complex properties. Therefore, there is no need to measure dynamic properties of the prepreg or laminate and the standard polymer characterisation testing suffices to have the input to the modelling.

## 2. Theoretical background

In this section, the theoretical formulation of the problem in different cases is reviewed together with the solution methods and possible approximations. Different problem cases used by different researchers for analysis of thermo-viscoelastic analysis of composites, are mentioned. The available formulation is used in the next section to present a general applied procedure for thermo-viscoelastic analysis of complex shaped (flat and curved) composite panels and structures.

### 2.1. Mathematical definition

The class of problems for which a solution procedure is presented in this paper, refers to anisotropic materials with a linear thermo-viscoelastic behaviour. The governing equations of the aforementioned boundary value problem can be formulated for a general three dimensional case. Later in the finite element implementation, this formulation is applied to plane, shell, 3D solid and 3D solid-shell elements for different modelling approaches.

– Three static equilibrium equations are:

$$\frac{\partial \sigma_{ji}}{\partial x_j} + F_i = 0 \tag{1}$$

– Linear strain–displacement equations are:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

In the above equations, $\sigma_{ij}$ is the component of the stress vector, $\varepsilon_{ij}$ is the component of the strain vector, $\rho$ is density and $F_i$ is the vector of inertial (body) force. Indices $i$ and $j$ stand for the number of degrees of freedom that is equal to 3 $(x,y,z)$ for a three dimensional case.

It is known that an orthotropic material has 9 independent stiffness components. For illustration of the components of stress, strain and stiffness, three dimensional constitutive equations for an elastic orthotropic material is written as [7]:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix}^{mech.} \tag{3}$$

$$\{\varepsilon\}^{mech.} = \{\varepsilon\}^{total} - \{\varepsilon\}^{thermal} - \{\varepsilon\}^{cure} - \{\varepsilon\}^{moisture}$$

Note that we will hereafter use the engineering Voigt notation for components of stress and strain like the left part of Eq. (3) in which stress/strain vectors, e.g. $\{\sigma_i\}, \{\varepsilon_i\}$, have 6 components.

In general, all the stiffness components and strains in Eq. (3) can be time and temperature dependent. However, the components in which the fibre properties are dominant may be assumed to be constant with time that simplifies the problem and reduces the computation time.

The nine time dependent stiffness components for an orthotropic material will be:

$$\begin{aligned} &C_{11} = \frac{1 - \nu_{23}\cdot\nu_{32}}{\Lambda}E_1(t); \quad C_{12} = \frac{\nu_{21} - \nu_{31}\cdot\nu_{23}}{\Lambda}E_1(t); \\ &C_{13} = \frac{\nu_{31} - \nu_{21}\cdot\nu_{32}}{\Lambda}E_1(t); \\ &C_{22} = \frac{1 - \nu_{31}\cdot\nu_{13}}{\Lambda}E_2(t); \quad C_{23} = \frac{\nu_{32} - \nu_{12}\cdot\nu_{31}}{\Lambda}E_2(t); \\ &C_{33} = \frac{1 - \nu_{12}\cdot\nu_{21}}{\Lambda}E_3(t) \\ &C_{44} = G_{23}(t); \quad C_{55} = G_{13}(t); \quad C_{66} = G_{12}(t) \\ &\Delta = 1 - \nu_{12}\nu_{21}(t) - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13} \end{aligned} \tag{4}$$

where Poisson's ratios are assumed to be constant with time. For a linear viscoelastic material it only matters how long $(t - \tau)$ it has been loaded, so the constitutive equation can be expressed as [1]:

$$\sigma_i(t) = \int_0^t C_{ij}(t-\tau)\frac{\partial \epsilon_j}{\partial \tau}d\tau \tag{5}$$

where $C_{ij}$ is the general relaxation stiffness matrix component.

Viscoelastic creep or relaxation response of a material depends strongly on temperature. In order to apply the effect of temperature, Eq. (5) should be modified as:

$$\sigma_i(t) = \int_0^t C_{ij}(T, t-\tau)\frac{\partial \epsilon_j}{\partial \tau}d\tau \tag{6}$$

The above formulation is for an isothermal problem in which the temperature does not change over time. For thermorheologically simple materials, the time–temperature superposition (TTS) principle [2] can be applied and the relaxation stiffness in Eq. (5) can be written as:

$$\begin{aligned} &C_{ij}(T(t), t) = C_{ij}(T_0, t_{red}) \\ &t_{red} = \int_0^t a_T(T_{(t)})d\tau \end{aligned} \tag{7}$$

In the above equations, $T_0$ is the reference temperature, $a_T$ is the time–temperature shift factor and $t_{red}$ stands for reduced time. As a result, the relaxation curve is obtained in a reference temperature and the response is calculated at a temperature $T$ for an arbitrary strain history. Therefore, scaling the time is enough to apply the effect of constant temperature as a change in the relaxation rate.