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Load carrying capacity of functionally graded columns with open cross-sections under static compression

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ABSTRACT

The nonlinear problems of static interactive buckling of thin-walled columns with a top hat cross-section and a lip channel cross-section, which are made of functionally graded materials (FGMs), are considered. The FG structures are subjected to static compression. The effect of temperature is neglected. It is assumed that functionally graded materials are subject to Hooke's law. An interaction of different modes has been analyzed in detail. Numerous different combinations of buckling modes have been computed. In all cases the theoretical value of load carrying capacity has been determined.

In order to obtain the equilibrium equations of thin-walled structures from Hamilton's Principle for the asymptotic analytical–numerical method. The classical laminate plate theory (CLPT) which has been modified in such a way that it additionally accounts for the full Green's strain tensor and the second Pioli–Kirchhoff's stress tensor has been applied. The study is based on the numerical method of the transition matrix using Godunov's orthogonalization. Distortions of cross-sections and a shear-lag phenomenon are examined. This paper is a continuation of the study described in the work of the authors entitled: "*Static interactive buckling of functionally graded columns with closed cross-sections subjected to axial compression*" Composite Structures 123, 2015, 257–262.

1. Introduction

Thin-walled structures subjected to compression or bending have many different buckling modes. Global buckling modes always lead to destruction of the structure. Although local buckling modes change the stiffness of the structure element, these types of structures are able to sustain load after local buckling. The most dangerous is the phenomenon of an interaction of global and local buckling modes, which significantly accelerates the process of structural damage. To determine the theoretical value of load carrying capacity, a modal interaction of buckling modes and imperfections has to be considered in the nonlinear post-buckling analysis. The investigation of buckling interaction requires an application of a nonlinear theory that enables estimation of an influence of different factors on the structure behavior.

The concept of interactive buckling (the so-called coupled buckling) involves the general asymptotic theory of stability [1]. Koiter's theory [2,3] is the most popular one, due to its general character and development, even more so after Byskov and Hutchinson [4] derived a complete set of formulas for the post-buckling constants associated with an interaction between modes. The theory is based on asymptotic expansions of the post-buckling path and is capable of considering nearly simultaneous buckling modes. The expression for potential energy of the system is expanded into a series relative to the amplitudes of linear modes near the point of bifurcation, which generally corresponds to the minimum value of critical load (the so-called bifurcation load). In the potential energy expression for the first order nonlinear approximation, the coefficients of cubic terms are the key terms governing the interaction. In the case the critical values corresponding to global buckling modes are significantly lower than local modes, their interaction can be considered within the first nonlinear approximation [5–9]. It is possible as the post-buckling coefficient for uncoupled buckling is equal to zero for the second order global mode in the Euler column model, and it is very often of little significance in the case of an exact solution. The theoretical load carrying capacity, obtained within the frame of the asymptotic theory of the nonlinear first order approximation, is always lower than the minimum value of critical load for the linear problem and the imperfection sensitivity can be obtained only.

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The consideration of displacements and load components in the middle surface of walls within the first order approximation, as well as precise geometrical relationships enabled the analysis of all possible buckling modes, including mixed buckling modes (e.g., the flexural–distorsional or local-distorsional mode – for a more detailed analysis, see [6,10–15]). In thin-walled structures of open cross-sections, due to their low rigidity, it is necessary to consider distortional deformations. The determination of the post-buckling equilibrium path requires the second order approximation to be taken into account. The numerical calculations presented in [16,17] have proven that an interaction of local modes having considerably different wavelengths is either very weak or does not occur at all. The expansion of Byskov and Hutchinson [4] is concerned with a number of interacting modes (local or global), but their examples are limited to one local and one global mode. In the paper by Byskov et al. [18], an interaction of global (overall) modes is treated with the Byskov and Hutchinson's method [4]. An interaction of buckling modes can occur among several buckling modes symmetric with respect to the symmetry axis of the cross-section and also between a symmetric mode and pairs of antisymmetric modes [6,19,20]. Dubina in [11] paid special attention to an interaction of global modes of buckling with a distortional and/or localized mode. An interaction of these buckling modes is strong according to the classification. In some cases, an improper selection of the mode, even if a few of them are considered, can lead to an overestimation of the structure load carrying capacity; also the consideration of the two-mode approach may sometimes be misleading and yield false conclusions. This task can be accomplished only by means of a nonlinear analysis [19,20]. Koiter and van der Neut [2] have proposed a technique in which an interaction of an overall (Euler) mode with two local modes having the same wavelength (i.e., three-mode approach) has been considered. The fundamental local mode is henceforth referred to as the "primary" one and the nontrivial higher mode (having the same wavelength as the "primary" one), corresponding to the mode triggered by the overall long-wave mode, is called "secondary".

Since the late 1980s, the Generalized Beam Theory (GBT) [10,21–23], pioneered by Schardt, has been developed extensively. Recently, a new approach has been proposed, i.e., the constrained Finite Strip Method (cFSM) [24–28]. These two alternative modal approaches to analyze the elastic buckling behavior have been compared in [29–31]. For the latest trends in the development of the GBT, see for example [32–38]. In these papers special attention is focused on an effect of global and local distortional modes on the load carrying capacity of structures.

In the literature, there are no studies on interactive buckling of functionally graded structures (the so-called FG structures). Some results have been published by the authors of this paper in [39,40]. In [39], the nonlinear Koiter's theory has been used to explain an effect of the imperfection sign (sense) on local post-buckling equilibrium paths of plates made of functionally graded materials (FGMs). In the case of the functionally graded square plate, nonzero first-order sectional inner forces that cause an occurrence of nonzero post-buckling coefficients responsible for sensitivity of the system to imperfections appear. It results in the fact that post-buckling equilibrium paths of plate structures made of FGMs are nonsymmetrically stable. This explains the differences in the plate response dependence on the imperfection sign (sense). The semi-analytical method (SAM) and finite element method (FEM) numerical results have been presented.

In [40], the static coupled buckling of thin-walled columns with trapezoidal and square cross-sections, which are made of functionally graded materials, have been considered. A three-mode approach has been adopted. In the case of interactive buckling of the structure with one axis of symmetry of the cross-section, the secondary local buckling mode has been a supplementary mode in the analysis of coupled buckling. Attention has been drawn to the effect of the imperfection sign (sense) and the nonsymmetric stable post-buckling equilibrium path on the load carrying capacity.

A comprehensive review of the literature concerning the interactive buckling analysis of structures can be found in [2,41]. Most of the papers describe the behavior of thin-walled columns made of isotropic, orthotropic or composite materials under mechanical loads. One can find some papers in which the uncoupled buckling behavior of thin-walled elements made of FGMs under compression (e.g., plates [42,43]) is presented. The nonlinear post-buckling analysis of this type of elements devoted to basic types of loads is covered in the monograph by Hui-Shen [44]. Also, thermal buckling and post-buckling analyses have been made for functionally graded thin-walled elements [45]. Due to the complexity of buckling problems of functionally graded plates under compound mechanical and thermal loads, the FEM is the only solution possible. Therefore, in the literature one can find many papers which present results of solutions to different problems of functionally graded structure buckling, obtained with an application of the FEM, for example [46–48].

2. Formulation of the problem

Thin-walled prismatic columns of the length l are considered. A plate model of the thin-walled structures has been applied. The boundary conditions for the column are taken as simply supported at their ends. The FG plates are made of an Al–TiC metal-ceramic material which is subject to Hooke's law [49]. The material properties are assumed to be temperature independent. In the present study, the classical laminate plate theory (CLPT) is used to obtain the equilibrium equations (Appendix A) [50]. In [51], Reddy has shown that the classical laminate plate theory is sufficiently accurate.

The geometrical relationships (i.e., full Green's strain tensor) are assumed in order to enable the consideration of both out-of-plane and in-plane bending of the plate [7,12,13,15,20,39–41,52,53]:

$$\begin{aligned} \varepsilon_x &= u_{,x} + \frac{1}{2}(w_{,x}^2 + v_{,x}^2 + u_{,x}^2) \\ \varepsilon_y &= v_{,y} + \frac{1}{2}(w_{,y}^2 + u_{,y}^2 + v_{,y}^2) \\ 2\varepsilon_{xy} &= \gamma_{xy} = u_{,y} + v_{,x} + w_{,x}w_{,y} + u_{,x}u_{,y} + v_{,x}v_{,y} \end{aligned} \quad (1)$$

and

$$\kappa_x = -w_{,xx} \quad \kappa_y = -w_{,yy} \quad \kappa_{xy} = -2w_{,xy} \quad (2)$$

where: $u,\ v,\ w$ – are components of the displacement vector of the plate in the $x,\ y,\ z$ axis direction, respectively, and the plane x—y overlaps the mid-plane before its buckling.

The nonlinear problem of stability has been solved with the asymptotic perturbation method. Let λ be a load factor. The displacement fields U and the sectional force fields N (Koiter's type expansion for the buckling problem) have been expanded into power series with respect to the dimensionless amplitude of the rth mode deflection ζ_r (normalized in the given case by the condition of equality of the maximum deflection to the thickness of the first component plate h_1) (see [2,41]):

$$\begin{aligned} U &\equiv (u, v, w) = \lambda U_0 + \zeta_r U_r + \zeta_r\zeta_q U_{qr} + \cdots \\ N &\equiv (N_x, N_y, N_{xy}) = \lambda N_0 + \zeta_r N_r + \zeta_r\zeta_q N_{qr} + \cdots \end{aligned} \quad (3)$$

where the pre-buckling (i.e., unbending) static fields are $U_0,\ N_0$, the first order nonlinear fields are $U_r,\ N_r$ (eigenvalues and eigenvectors problems), and the second order nonlinear fields – $U_{qr},\ N_{qr}$, respectively. The range of indexes is $[1,J]$, where J is the number of