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Layered orthotropic plates. Generalized theory

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ABSTRACT

In this paper a two-dimensional layer-wise generalized theory of layered orthotropic plates is presented. This theory is based on a power series expansion of the displacement vector component in each layer for the transverse coordinate. The number of terms retained in the power series is arbitrary and it is chosen depending on the problem being considered.

The potentialities of the offered approach are demonstrated by examples of investigating the response of composite plates to static and dynamic loading. Also, the feasibility of the theory is shown by example of the problem of a ball impacting a composite plate.

The stress–strain state of multilayer composites has been considered for distributed and space-localized loads. The results of calculations according to the generalized theory are compared with exact solutions given Pagano, as well as with data obtained from the two-dimensional theories.

1. Introduction

Orthotropic layered structures are used widely in modern construction, transport machinery, aerospace industries due to their excellent properties. So, over the past decades a significant growth of use of composite materials in airplane construction has been observed. High-quality design of orthotropic layered structures is impossible without effective mathematical models and methods to assess their strength.

In the theory of layered structures there exist two approaches to building two-dimensional theories which are called layer-wise and equivalent single-layer [1–4]. The layer-wise approach uses hypotheses for each separate layer to derive equations [4–8]. The second approach uses hypotheses for the pack as a whole to derive the governing equations [4,5,9,10]. Each approach has its advantages and deficiencies, but it is believed that layer-wise approach makes it possible to obtain a more accurate description of strains and stresses in each pack layer, as well as of the interlayer contact conditions. A review of the layer-wise and equivalent single-layer theories contains in works Reddy [1], Noor and Burton [11], Grigoliuk and Kogan [12], Chen and Wu [13], Carrera [14], Mallikarjuna and Kant [15]. The key feature of the modern stage of development of the mechanics of layered structures consists in the transition from more simple models to more complex ones possessing higher accuracy and universality.

A fair number of different methods of layered structure design have been developed to date [16–25]. Classical theories based on the straight normal hypothesis have been found inapplicable in the majority of cases for layered composite structures [18,21,23,25]. Hence, refined theories of layered composites have become widespread [16–25]. The refined theories based on various variant on the broken line hypothesis or zig-zag functions are applied primarily for analysis of strain–stressed state (SSS) of layered plates [7,14,18,19]. And other approaches have been developed in recent years. Ferreira et al. [17] used trigonometric shear deformation theory to modelling symmetric composite plates. Carrera [6,14,17,26] suggested unified formulation for analysis multilayered plates and shells. Demasi proposed generalized unified formulation theory of layered plates which is an extension of Carrera's unified formulation [27].

One of the methods of building refined models is applying expansions into series of transverse coordinate displacements [1–4,13,21,22]. These are power series and orthogonal polynomial expansions. Polynomial expansions yield more precise approximations. However, the algorithm in this case is more complex, and only the first terms of these expansions can be taken into account in the resolving equations.

Two-dimensional models can be built readily by using expansions into power series of transverse coordinate displacements [1–4,13,21,22]. The power series method has found its application in both layer-wise and equivalent single-layer approaches. In work [28] has shown that an increase in the number of retained terms in power series leads to an increase in the model's accuracy, which

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ultimately leads to an extension of model application limits. However, as a rule, the first order theories of layered plate are applied primarily for analysis of SSS of composites [4,7,19,24,25]. The first-order theories proceed from that strain is a linear function of the transverse coordinate. In this case, the transverse strain is ignored as a rule or considered invariable over the layer thickness.

It has been known that, with increasing structure thickness or local loading (the characteristic size of the loading area is comparable with plate thickness), an essentially nonlinear pattern of distribution of stresses and strains over thickness is observed [8,21,28–31].

The majority of studies have dealt with describing the SSS of multilayer structures under static loading [1–6,9,19,21–25,29,30]. They have considered in detail the issues of applicability of different theories for SSS design in structures with different geometric characteristics under the effect of distributed loads when the characteristic size of the loading area is commensurable with characteristic plate dimensions. However, the issues of modelling the SSS of multilayer composites subjected to localized loads remain to be adequately investigated.

The number of studies dealing with investigating the behaviour of multilayer orthotropic plates under dynamic loading is far less [25,32–35]. These problems are more challenging and often demand the use of refined models. This is why the number of studies in this area is limited, and their majority is dedicated to investigating the response of structures to distributed impulse loads. The number of publications dealing with investigating the dynamics of such objects under localized loading is far less.

Transverse impact with a solid body is especially hazardous for composites. This yields a local loading pattern with short-time action. Certain problems of modelling the response of composites to low-velocity impact with a solid body were discussed [36–40]. To model composite behaviour, Kirchhoff's plate theory [40] and first-order refined theories (of the Timoshenko type) [36–38] were used. They failed to account for reduction and the nonlinear pattern of stress distribution over the thickness. Study [37] has offered a solution with the analytical method, and papers [38,39] have used the finite-element method.

This paper describes the development of the generalized theory of multilayer plates with isotropic layers [28,41] for the case when the layers are made of an orthotropic material. The approach suggested is based on presenting displacements in each layer as a power series for the transverse coordinate. This makes it possible to compute displacements and stress tensor components in each layer with a specified accuracy, and meet the conditions of contact of adjacent layers. This two-dimensional approach is a generalization of the class of layer-wise theories, which are based on an expansion of displacements in power series and can be thought of as a variant of unified formulation of displacement based theories [26].

2. Problem statement and governing equation

2.1. Problem statement

A constant-thickness layered plate is referred to the Cartesian system of coordinates $Ox_1x_2x_3$ associated with the external surface of the first layer (Fig. 1). The contact between layers is assumed to exclude their delamination and mutual slipping. The number of layers in a pack is I and h_i is thickness of the ith layer. The layers are made of orthotropic materials, and θ_i is reinforcing angle in the ith layer. The reinforcement directions in each layer are assumed to be parallel to coordinate axes Ox_1, Ox_2.

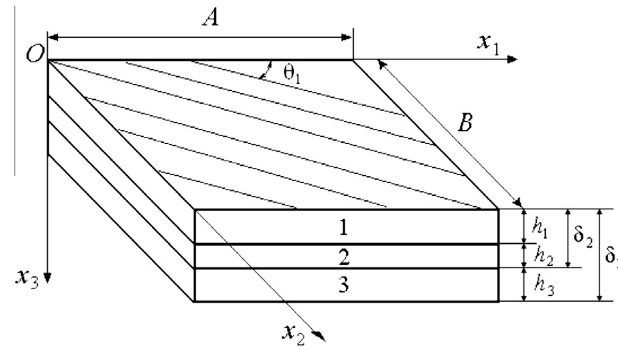

Fig. 1. Layered orthotropic plate.

External force $\bar{q} = \bar{q}(x_1, x_2, t)$ is applied to the external surface of the first layer. Let q_α denote the projection of the external load vector on the coordinate axis.

The behaviour of the layered plate is described by equations of the generalized theory of multilayer plates [28,41]. The theory allows selecting the required accuracy of describing SSS depending on the composition of the pack of layers. In general, the displacement of a point on the ith layer is described by the following kinematic relationships:

$$u_\alpha^i(x_1, x_2, x_3, t) = u_\alpha + \sum_{j=1}^{i-1}\sum_{k=1}^{K_\alpha^j} h_j^k u_{\alpha k}^j + \sum_{k=1}^{K_\alpha^i}(x_3 - \delta_{i-1})^k u_{\alpha k}^i,$$

where $h_j^k = (h_j)^k, \delta_i = \sum_{j=1}^i h_j, \delta_{i-1} \leqslant x_3 \leqslant \delta_i, i = \overline{1,I}$;

u_α^i $(\alpha = \overline{1,3})$ is displacement of an ith layer point in the direction of axis $Ox_\alpha; u_\alpha,\ u_{\alpha k}^i$ are coefficients of expanding displacements into power series, which are functions of arguments $x_1,\ x_2,\ t; K_\alpha^i$ are maximum exponents of the transverse coordinate for plane $(\alpha = 1,\ 2)$ and transverse $(\alpha = 3)$ displacements of the ith layer.

To simplify and improve problem algorithmicity, we will take into account the same number of terms of the power series in all layers, i.e. $K_1^i = K_1, K_2^i = K_2, K_3^i = K_3$ $(i = \overline{1,I})$, where $K_1,\ K_2,\ K_3$ are parameters specified depending on the required accuracy of problem solution. In this case, the generalized theory hypotheses can be simplified and written in more compact form

$$u_\alpha^i(x_1, x_2, x_3, t) = u_\alpha + \sum_{k=1}^{K_\alpha}\left[\sum_{j=1}^{i-1} h_j^k u_{\alpha k}^j + (x_3 - \delta_{i-1})^k u_{\alpha k}^i\right], \qquad (1)$$

In so doing, K_1 and K_2, which describe the number of retained power series terms for plate displacements, will be the same and equal to K. Henceforth, the generalized theory shall be designated by the number of retained terms in power series (1) for plane and transverse displacements – theory $\{K, K_3\}$.

The accepted kinematic relations (1) for $K = 1,\ K_3 = 0$ are equivalent to the hypotheses in the theory of Grigoliuk and Chulkov [19,41], at $K = 1,\ K_3 = 1$ they are equivalent to the hypotheses of the refined first-order theory [7] and at $K = 3,\ K_3 = 2$ they are equivalent to the hypotheses of the refined high-order theory [8,41].

The strain in each plate layer is supposed to be small and it is described by linear relationships

$$\varepsilon_{\alpha\beta}^i = \frac{1}{2}\left(u_{\alpha,\beta}^i + u_{\beta,\alpha}^i\right), \quad \alpha = \overline{1,3}, \quad \beta = \overline{1,3}, \quad i = \overline{1,I}.$$

With account of the accepted hypotheses (1), the components of strain tensor $\varepsilon_{\alpha\beta}^i$ take the form