Composite Structures 131 (2015) 565–573

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

An investigation of matrix damage in composite laminates using continuum damage mechanics

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ARTICLE INFO

Article history:
Available online 10 June 2015

Keywords:
Polymer-matrix composites
Matrix cracks
Continuum damage mechanics
Damage evolution
Numerical simulation

ABSTRACT

Transverse and shear damage in the polymer matrix of fiber reinforced composites are often treated independently, while the associated cracks are similar in appearance and effect. Matrix damage is often quantified by its crack density. However, damage also can be presented as a state variable. The definition of damage state variables and their evolution laws are the foundation of continuum damage mechanics, yet experimental data for shear damage evolution is limited. The effect of crack closure on the transverse stiffness of laminates was already incorporated in the damage models. However, the shear modulus sensitivity to crack closure has received little attention due to the complicated constitutive equations. This paper focused on the in-plane shear modulus reduction due to matrix transverse cracks and the evolution of transverse and shear damage.

The comparison of existing models with experimental data showed that shear damage is currently not suitably described. Accordingly, a shear damage evolution model was proposed that included the effect of internal traction and crack closure on the in-plane shear modulus. The numerical predictions of the modified shear model resulted in good agreement with experimental data by predicting the same saturation crack density, axial and shear responses.

1. Introduction

While fiber reinforced laminate applications in the aircraft industry are rapidly increasing, there is a lack of accurate failure prediction and progressive damage analysis. The efficient design of a composite structure depends on developing accurate analytical and numerical models. Critical to this advancement is a thorough understanding of damage mechanisms and their interactions.

Laminate progressive failure models depend on a failure criterion for the first ply failure and a material degradation model to describe hardening or softening after first ply failure. Failure in the fiber direction is usually catastrophic, while degradation factors are applied when failure occurs in the matrix. The classical approach toward material degradation modeling in fiber reinforced composite materials describes material stiffness as a function of crack density. There are different analytical, semi-analytical and numerical models which focus on these solutions [1,2]. Another approach describes damage as a macroscopic state variable and defines material degradation in the framework of continuum damage mechanics [3]. In fiber reinforced composites, matrix damage can be described as cracks either parallel or perpendicular to the fiber direction. Accordingly, a tensor can be used to represent damage with principal directions aligned with the material directions.

Ladeveze and La Dantec [4], were among the first to apply a damage mechanics model to composite laminates. In this model, damage mechanics was used to describe the matrix microcracking and fiber–matrix debonding. Ladeveze used different damage variables in the shear and transverse directions. Engblom et al. [5] used the same damage variable in all directions. They used the Hashin criterion and assumed a linear relation between shear and transverse damage. Barbero et al. [6,7] proposed a damage model based on a second order damage tensor. A new expression for a damage surface was proposed, which reduced to the expression of the Tsai-Wu failure criterion in stress space. It was assumed that the lamina material directions coincide with the principal damage directions for a lamina. Therefore, the second order damage tensor can be characterized as a diagonal matrix, where damage variables are the eigenvalues of the damage tensor, which represent the damage ratio along the principal directions. Nuismer and Tan [8] adopted the shear lag model to find effective in-plane compliance relations of a matrix cracked lamina. They defined transverse and shear damage as internal state variables representing the lamina damage state directly.

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http://dx.doi.org/10.1016/j.compstruct.2015.06.012
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Matzenmiller et al. [9] proposed a fourth order damage tensor. Damage variables described the dominant failure modes. The model included modified evolution laws for damage with respect to interaction terms. Maimi et al. [10,11] modified the damage stiffness tensor and evolution laws which were proposed by Matzenmiller et al. [9], while the compliance tensor was the same. However, Maimi et al. [10] derived their constitutive model based on the thermodynamics of irreversible damage in a polymer composite.

Transverse stiffness reduction has been widely studied, where agreement between models and experiment is common. By comparison, shear modulus softening from matrix cracks has received little attention, and tends not to agree with prediction. The laminate damage models including models mentioned above [4–11] assume linear relation between transverse and in-plane shear damage without experimental evidences. According to the linear assumption the matrix cracks parallel to the fiber direction have the same effect on the transverse and shear modulus. However the experimental investigations by Knops et al. [12] and later by Smith and Salvatian [13] showed that the shear modulus was less affected than the transverse modulus. These experimental observations were the motivation of the current study. This work focused on matrix cracks parallel to the fiber due to loading perpendicular to the fibers, (hereafter referred to transverse loading) and considered the effect of matrix cracks on transverse and shear damage evolution. The accuracy of the linear relation between shear and transverse damage was investigated in this study. The results were compared with existing models [4–11], which over predicted the observed in-plane shear modulus reduction.

The objective of this work was the development of an improved shear damage evolution model for fiber reinforced laminates. A model was implemented in Abaqus using a UMAT subroutine. The crack band theory [14] was included in the model to eliminate the mesh dependency of the model.

2. Continuum damage model

From a practical point of view, a difficulty of the damage models for composite materials is the number of parameters needed to characterize their anisotropic behavior. To model the damage response of a fiber reinforced lamina, most continuum damage models define two independent damage variables according to the common failure modes in a lamina. In a fiber reinforced lamina d_1 represents damage in the fiber direction, whereas the d_2 represents damage in the matrix perpendicular to the fiber direction. When $d_i = 0$ there is no damage or stiffness loss of the element while $d_i = 1$ means total failure of the element and zero stiffness. Obviously, it is possible to define independent damage variables in shear direction and also introduce independent variables for the interactions between damage modes. However, given the number of tests needed to define the unknown parameters, as the number of variables increases, the likelihood of experimental verification decreases.

The current proposed damage model is based on independent orthogonal damage orientations in a lamina coinciding with the material directions. Damage in the shear direction is defined as functions of the independent damage variables. We assume, therefore, that transverse matrix cracks influence the transverse and shear response, regardless of how the transverse damage was induced. Since this work only considered matrix cracks, shear damage was defined as a function of transverse damage which simplified the formulation for this demonstrative application. The formulation is in the local material coordinate system for each lamina. The constitutive law of the proposed damage model is based on the complementary free energy density and the second law of the thermodynamics. In this approach, it is assumed that there is a positive definite potential which is a function of free variables (the stresses) [15]. The positional function must enclose the origin and become zero at the origin with respect to the free variables [10]. The proposed form of the Gibbs energy is based on the works of Maimi et al. [10,11] and Lapczyk et al. [16] including the residual terms introduced by Chaboche and Maire [17] as

$$
\begin{aligned}
\phi = {} & \frac{1}{2E_1(1-d_1)}\sigma_{11}^2\xi(\sigma_{11}) + \frac{1}{2E_1}\sigma_{11}^2[1-\xi(\sigma_{11})] \\
& + \frac{1}{2E_2(1-d_2)}\sigma_{22}^2\xi(\sigma_{22}) + \frac{1}{2E_2}\sigma_{22}^2[1-\xi(\sigma_{22})] - \frac{\upsilon_{12}\sigma_{11}\sigma_{22}}{E_1} \\
& + \frac{(\sigma_{12}-\sigma_{12}^r)^2}{2G_{12}(1-d_2)}\xi(\sigma_{22}) + \frac{(\sigma_{12}-\sigma_{12}^r)^2}{2G_{12}(1-\beta d_2)}[1-\xi(\sigma_{22})] \\
& + \phi^r(\sigma_{12}^r, d_2, \beta, \xi(\sigma_{22}))
\end{aligned}
\tag{1}
$$

where, ϕ^r is the residual energy density (defined below) and σ_{12}^r is the residual shear stress (which is takes the value 0 as an initial value). The step function $\xi(x)$ is defined as

$$
\xi(x) = \begin{cases} 0, & x \leqslant 0 \\ 1, & x > 0 \end{cases} \tag{2}
$$

As will be discussed below, traction is observed to act across the crack faces under in-plane shear loading. This gives rise to a residual stress when a material with damage if loaded and loaded in shear. In compression, the proposed Gibbs energy function assumes no change in the transverse or axial stiffness, even in the presence of existing damage. (In general the proposed model can account for compression damage by defining damage variables for the compression energy density). It was assumed the in-plane shear modulus of the damaged material depended on the sign of the transverse stress. A material dependent parameter (β) described the sensitivity of the in-plane shear modulus of the damaged material to transverse stress. To simplify the formulation, and since the damage model and experimental investigation focused on matrix damage, shear damage was defined only as a function of transverse damage. The proposed approach can, nevertheless, describe shear damage as a function of fiber damage or any other independent damage mode. The strain tensor was derived using the proposed complementary free energy density function as

$$
\varepsilon = \frac{\partial\phi}{\partial\sigma} = S : \sigma \tag{3}
$$

where S is the material compliance matrix.

2.1. Residual energy formulation

There are many continuum damage models based on damaged material response. However, only a few models can simultaneously simulate material anisotropy and the effect matrix crack closure on the lamina stiffnesses. The main challenge is introducing a material model which correlates with experimental evidence and satisfies the second law of thermodynamics. Chaboche and Maire [17] proposed a theory for dissipative behavior of ceramic matrix composites which considered crack closure effects and the corresponding energy function. They used a set of scalar damage variables and a fourth order damage tensor to describe the damage evolution in ceramic matrix composites. The same concept was adopted here using a second rank damage tensor. Chaboche and Maire [17] assumed undamaged shear modulus due to crack closure, while the current model assumes reduction in shear modulus even when the cracks are closed but with a different rate compared with the open crack mode. In the shear direction, crack closure was modeled by introducing an extra term in the complementary free energy definition. Considering the energy density function, Eq. (1), and