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An analytical joint stiffness model for load transfer analysis in highly torqued multi-bolt composite joints with clearances

Fengrui Liu [a], Jianyu Zhang [b,*], Libin Zhao [a], An Xin [a], Longwei Zhou [a]

[a] School of Astronautics, Beihang University, Beijing 100191, PR China
[b] College of Aerospace Engineering, Chongqing University, Chongqing 400044, PR China

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ABSTRACT

An analytical tri-linear joint stiffness model accounting for the effect of bolt-hole clearances on the bearing chord stiffness is presented. A single-parameter iteration method is also proposed to solve the nonlinear system of equations of the load transfer analysis, in which the recursive formulae are verified to be monotonic. To verify the present joint stiffness model, a series of single-lap, single-bolt joints with various bolt-hole clearances and bolt tightening torques were tested to empirically confirm the influence coefficients of the joint stiffness model. The load distributions in three single-lap, three-bolt joints with various clearances were also tested under finger-tight and highly torqued conditions. The load distributions in multi-bolt joints predicted by the presented joint stiffness model together with the single-parameter iteration method are in good agreement with the experimental outcomes, verifying the accuracy of the presented joint stiffness model and the feasibility of the single-parameter iteration method. In addition, the effects of the friction forces originated from the bolt-tightening torque and bolt-hole clearances on the load distribution in multi-bolt joints are presented.

1. Introduction

Multi-bolt composite joints are the preferred connections in composite structures because they are able to be dismantled and detected conveniently and have high load-carrying and load-transferring capacity. However, as a core component to transfer high loads, the multi-bolt composite joint is a weak spot of composite structures, and its strength requires further attention [1,2]. The failure prediction of multi-bolt composite joints includes two steps, an overall joint analysis to determine the load distribution between bolts and a detailed local strength analysis around a critical hole to ensure a safety margin for all failure modes [3–5]. Due to the anisotropy and brittleness of composites, multi-bolt composite joints experience a minimal amount of plastic stress relief and mechanical softening compared to their metallic counterparts, which leads to an uneven load distribution between bolts. Each bolt hole in the joints is subjected to differently combined bearing bypass loads. Because the final catastrophic failure of the joints is generally determined by the combined bearing bypass stresses at the critical hole, it is important to conduct an accurate load transfer analysis in multi-bolt composite joints for determining the combined stress status of the critical hole.

Many factors must be considered in the load transfer analysis of multi-bolt joints, including joint materials, laminate lay-ups, laminate thickness, hole diameter, fastener position (ratios of width-to-hole diameter and edge-to-hole diameter), fastener type (countersunk or protruding head), bolt-hole clearance and bolt-tightening torque. Existing analytical approaches including the complex potential method [6,7], specification of displacement expressions [8] and complex stress functions [9] are deficient in accounting for the effects of the fastener type, bolt-hole clearance and bolt-tightening torque. Although finite element analysis (FEA) [10–13] can capture the influences of all of the factors, it suffers from the need for professional complex modeling techniques and excessive computational time, especially when friction and damage models are incorporated [14]. The load transfer analysis in multi-bolt joints is crucially dependent on the joint stiffness, which is mainly dominated by the stiffness distribution among the laminates and bolts as well as their contact status. By idealizing the multi-bolt joint as a series of springs and masses and further assigning a stiffness value to each component in the joint, simple spring-based methods [14–18] were presented and have been widely used in engineering to calculate the load distribution in multi-bolt joints with high efficiency and at low cost.

In simple spring-based methods [15,16], the joint stiffness reflects an assumed linear variation of the bolt load with the hole deformation, as illustrated by an oblique line passing through the

* Corresponding author.
E-mail addresses: jyzhang@cqu.edu.cn, jyzhang@buaa.edu.cn (J. Zhang).

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coordinate origin on the load-deformation graph of a single-bolt joint. Accounting for a delay in the load take-up caused by the bolt-hole clearance in joints, Hart-Smith [17] developed a joint stiffness model by shifting the oblique line horizontally with a clearance off-set. Further experimental investigations on highly torqued single-bolt joints with bolt-hole clearance by McCarthy and Gray [14] indicated a stiffer connection nature before the delay in the load take-up because the joint load was reacted to by static friction forces between laminates. Based on experimental studies, they proposed a tri-linear model for the joint stiffness, including an initial quasi-linear region with *Slope* 1, a horizontal transition region and a linear region with *Slope* 2. All parameters in McCarthy and Gray's [14] joint stiffness model, including an *initial stiffness* in the initial quasi-linear region, a *bearing chord stiffness* in the linear region and a *critical friction load* in the transition region, were calculated using simplified formulae, which streamlined the load transfer analysis. The model covered the static friction forces originating from the tightening torque and the clearance of *Slope* 1 and the transition region while ignored the effects of the clearance of *Slope* 2. However, the influence of the clearance on the bearing chord stiffness was clear based on the experimental load–displacement curves of double-lap single-bolt joints with different clearances [19,20]. The bearing chord stiffness of *Slope* 2 determines the load participation coefficient of bolts, and the linear region dominates the entire bolt load transfer history. Thus, the influence of the clearance of *Slope* 2 must be considered.

The load–displacement curve of the developed joint stiffness model presents a piecewise linear characteristic. This means that the system of equations obtained by using free body diagrams together with equations of motion under quasi-static loading for each mass must be solved by iterative methods. As the number of bolts and degree of complexity of the nonlinear joint stiffness model increase, the time and operations needed to solve the system of equations increases significantly [14,17].

The objective of this paper is to establish a joint stiffness model that accounts for various bolt-hole clearances and bolt-tightening torques for load transfer analysis in multi-bolt joints. A simple and clear single-parameter iteration method is also presented to solve the system of equations in load transfer analyses. Four parameters in the proposed model were empirically confirmed by using single-lap, single-bolt joints with different bolt-hole clearances and tightening torques. With the proposed joint stiffness model and single-parameter iteration method, the load distributions in single-lap, three-bolt joints with finger-tight and 7 N m tightening torque as well as varying clearances were calculated. Experiments on the load distributions in multi-bolt joints were also performed to verify the effectiveness of the proposed model.

2. A developed joint stiffness model and single-parameter iteration method

A tri-linear model accounting for varying clearances and tightening torques is presented to describe the joint stiffness and calculate the load distribution in multi-bolt joints. Furthermore, a single-parameter iteration method is presented for solving the system of equations of load transfer analysis in multi-bolt joints. This method depends on a series of recursive formulae with only one unknown, which is proposed by using a spring-beam model of multi-bolt joints and verified to be monotonic.

2.1. Single-parameter recursive formulae

A spring-beam model for a multi-row, single-lap composite joint is depicted in Fig. 1, in which the laminates, ignoring the segments between Bolt 1 and the left free-end of laminate as well as that between Bolt n and the right free-end of laminate, are idealized as series of springs and the bolts are assumed to be a group of beams. The external tensile loads (denoted by F) are directly applied to the outermost bolts. F_i and δ_i ($i = 1,\ldots,n$) indicate the bearing load on the ith bolt and the longitudinal deformation (along the loading direction) corresponding to the ith bolt, respectively. $F_{i,\mathrm{A}}$ and $F_{i,\mathrm{B}}$ indicate the inner forces of the laminates between the ith and $(i+1)$th bolts, in which subscripts A and B denote the member laminates A and B, respectively.

Similar laws can be found for Part a, b and c in the spring-beam model, from which concise recursive equations can be derived. From the free-body diagram of Part a, the equation group for the first bolt is

$$\begin{cases} F_1 = f(\delta_1) \\ F_{1,\mathrm{A}} = F - F_1 \\ F_{1,\mathrm{B}} = F_1 \end{cases} \quad (1)$$

where function $f(\delta_1)$ represents the joint stiffness model.

Similarly, from the free-body diagram of Part b, the longitudinal deformation, the bearing load on the $(k+1)$th bolt and the inner force of the laminates can be described using the parameters of the frontal bolt as follows:

$$\begin{cases} \delta_{k+1} = \delta_k + (F_{k,\mathrm{B}} + F_k)/k_\mathrm{B} - F_{k,\mathrm{A}}/k_\mathrm{A} \\ F_{k+1} = f(\delta_{k+1}) \\ F_{k+1,\mathrm{A}} = F_{k,\mathrm{A}} - F_{k+1} \\ F_{k+1,\mathrm{B}} = F_{k,\mathrm{B}} + F_{k+1} \end{cases} \quad (2)$$

where k_A and k_B are the equivalent stiffnesses of the member laminates A and B, respectively. They can be calculated by the formula [14] $k_j = E_{\mathrm{L}j} W_j t_j / L_p$ (j = A or B), where $E_{\mathrm{L}j}$, W_j and t_j are the longitudinal homogenized Young's modulus, width and thickness of the member laminates A and B, respectively. L_p is the bolt pitch.

Recursively, the longitudinal deformation and bearing load on the nth bolt can be expressed by those of the $(n-1)$th bolt, referring to Part c.

$$\begin{cases} \delta_n = \delta_{n-1} + (F_{n-1,\mathrm{B}} + F_{n-1})/k_\mathrm{B} - F_{n-1,\mathrm{A}}/k_\mathrm{A} \\ F_n = f(\delta_n) \end{cases} \quad (3)$$

Equations (1)–(3) which have unique unknown δ_1, are single-parameter recursive formulae for the load transfer analysis in multi-bolt joints. The bolt bearing loads F_i ($i = 1,\ldots,n$) can be calculated recursively if the joint stiffness model $f(\delta)$ is available, which will be discussed in detail in the following subsection.

2.2. Joint stiffness model accounting for bolt tightening torques and bolt-hole clearances

A typical empirical joint stiffness model described by the load-deformation curve of a single-lap, single-bolt composite joint is shown in Fig. 2(a). Three representative stages corresponding to the initial quasi-linear region, transition region and bolt load transmittal region can be observed, in which the relative locations of the bolt and laminates are depicted in Fig. 2(b)–(d), respectively.

During the static test of a single-lap, single-bolt joint, the external tensile load applied to the joint was initially reacted to by the static friction forces between the laminates as well as between the bolt and the laminates, stemming from the bolt-tightening torque. Without initial slippage between the laminates, the small hole deformation was mainly caused by the shear deformation of the laminates. Thus, the joint stiffness was primarily dominated by the stiffness of the laminates, which led to the highest slope in the initial quasi-linear region. With the increase in the external load, the static friction forces were overcome, and the laminates began to slip relative to each other. During this phase, the joint