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Numerical evaluation of effective elastic properties of composites reinforced by spatially randomly distributed short fibers with certain aspect ratio

Lehua Qi *, Wenlong Tian, Jiming Zhou

School of Mechanical Engineering, Northwestern Polytechnical University, 127 Youyi West Road, Xi'an 710072, PR China

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ABSTRACT

The effective elastic properties of composites reinforced by spatially randomly distributed short cylindrical fibers with certain aspect ratio are investigated using the numerical homogenization method. A modified RSA algorithm is proposed to generate the periodic RVEs. The periodic boundary conditions are introduced and the periodic RVEs thus created are analyzed to obtain the mechanical properties of composites by using the FE package ABAQUS. The ABAQUS Python Interface is used to introduce the periodic boundary conditions and to obtain the average stresses and strains of RVEs. The simulation results show that the periodic boundary conditions guarantee the continuity of strain and stress fields on the boundaries of RVEs. In the case of the fiber aspect ratio of 15 and fiber volume fraction of 10%, it is sufficient to consider the size of RVE as $L/l = 2.5$, in which an approximately random fiber orientation exists. The effective elastic properties of composites obtained by the numerical homogenization agreeing well with those obtained from the traditional equations for composites based on the Halpin–Tsai estimation and with those measured from the uniaxial tensile experiments shows the validation of the numerical homogenization for effective elastic properties of composites reinforced by spatially randomly distributed short fibers.

1. Introduction

Short-fiber reinforced composites, especially short-carbon-fiber reinforced magnesium matrix composites, are of paramount importance in the automotive, aviation, aerospace and national defense industries, etc., [1,2], because of their sound lightweight, excellent mechanical and physical properties [3,4], which claims an essential requirement of the accurate mechanical properties characterization to expand their applications. Owing to the time-consuming and costly testing of the experiments, an alternative, or at least a complement to experiments, is the analytical or numerical prediction on the mechanical properties of short fibers reinforced composites, which have been developed rapidly in the last decades. The main difficulties in predicting the macroscopic mechanical constitutive behaviors of inhomogeneous composites lie in relating it to the micro-scale inhomogeneity of the constituents [5].

Traditionally, the analytical and semi-analytical homogenization models have been used to evaluate the effective mechanical properties of composites. The analytical homogenization models based on the Eshelby's tensor [6] to predict the constitutive elastic behaviors of composites, where a fiber was treated as a second phase inclusion, enjoyed considerable support. The significant success had been appreciated by the techniques based on the Mori–Tanaka (MT) mean-field method [7], which hypothesized that within the composites with many identical inclusions, each inclusion experienced a far-field strain which equaled to the average matrix strain. The MT model had gained substantial interest, particularly following the clarifications provided by Benveniste et al. [8] and the methods application to composites [9,10]. And the generalization of the MT method provided by Weng et al. [11] and the solution for the complete set of the elastic materials properties [12] guaranteed the methods popularity for the analysis of composites. However, the aforementioned analytical homogenization models based on the Eshelby's tensor originally were developed and used to investigate the effective elastic properties of composites with the aligned inclusions (for example fibers). If these homogenization models were employed to determine the mechanical properties for the random fibers reinforced composites, the fiber orientation averaging, as developed by Advani and

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* Corresponding author. Tel.: +86 29 88460447; fax: +86 29 88491982.
E-mail address: qilehua@nwpu.edu.cn (L. Qi).

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Tucker [13], had to be performed in order to account for the random fiber orientation [14].

A different set of methodologies for the characterization of mechanical responses of the random fiber reinforced composites was based on the classical laminated plate theory (CLPT), as illustrated by Laminated Random Strand Method proposed by Ionita and Weistman [15] for the analysis of the high volume fraction carbon fiber/urethane composites. According to the assumption that the strand orientation was predominantly parallel to the plane of the plate considered, the material was idealized as a set of parallel layers of in-plane randomly oriented fibers. Empirically accounting for strand overlap, the CLPT approach was used to compute the global stiffness matrices of composites.

By the availability of powerful computational hardware and advanced FE packages, a third set of the computational homogenization methods for short fiber reinforced composites have been an active area of research starting with a seminal contribution of Guedes and Kikuchi [16] for linear elasticity problems, in which a generic finite element based approach by solving the boundary value problem for the effective modulus was demonstrated and different shapes of inclusion were used, varying from whisker to platelet.

However, few considerations are given to the micro-mechanical stress and strain state in the aforementioned theoretical methods and semi-empirical theories, thus the finite element analysis is widely employed. The representative volume elements (RVEs) combined with the finite element method (FEM) [17,18], is proven to be a powerful technique, which can consider a more complex microstructure given by several inclusions with the different shapes, different orientations and different aspect ratios, and even the random distribution of inclusions. For this RVE based FEM method, there is no restriction on the geometry, on the material properties, and on the number of phases in the composites, which results that the RVE based FEM has been well documented for the determination of the effective material properties of short fibers reinforced composites. Kari et al. [19] employed FE analysis to investigate the effect of volume fraction, fiber aspect ratio on a random short SiC-fiber reinforced metal matrix composite (MMC). Bohm et al. [20] and Duschbauer et al. [21] studied a MMC reinforced by approximately planar random short fibers. Note that the key issues of the RVE based FEM approach are identification and generation of an RVE through which the effective homogeneous material properties are derived, and the application of the periodic boundary conditions.

In this article, the emphasis is on the evaluation of effective elastic properties of composites reinforced by spatially randomly distributed fibers using the numerical homogenization techniques (RVE based FEM). To obtain the realistic predictions of the macroscopic mechanical behaviors of composites reinforced by spatially randomly distributed fibers, periodic RVEs are generated using the random sequential adsorption algorithm [22,23] and the numerical homogenization of statistical RVEs are performed. Several investigations are conducted in order to determine the critical size of RVEs of such composites with certain fiber aspect ratio. A set of statistical analyses is performed to determine the mean values of the mechanical properties coefficients, as well as measures of isotropy.

2. Homogenization of effective elastic properties of composites

Numerical homogenization, main idea of which is to find a globally homogeneous medium equivalent to the original composites, where the strain energy stored in both systems is approximately same, is an excellent method to predict the effective elastic constants of the heterogeneous materials on the micro-scale.

2.1. Generation of representative volume element

To implement the numerical homogenization of effective elastic properties of composites, the first and important step is to generate periodic RVEs which can characterize the microstructure of composites. The homogenized effective elastic properties of composites are obtained by the finite element analysis of the periodic RVEs consisting of the spatially randomly distributed short non-overlapping fibers.

For the periodic RVEs, if any surface of fiber cuts any of RVE surfaces, this condition has to be checked with the fiber volumes on the opposite surfaces because of periodicity. Furthermore, the surfaces of all fibers should not be very close to the surfaces as well as to the corners of cubic RVE in order to avoid the presence of the distorted finite elements and obtain the regular mesh within the opposite surfaces, which is convenient to apply the periodic boundary conditions.

Here, the modified random sequential adsorption algorithm [22,23] combining the above conditions is used to generate the periodic RVEs of composites reinforced by the spatially randomly distributed fibers up to the desired volume fractions and the flow chart is given in Fig. 1. In general, these algorithms can generate up to 15% volume fractions with the identical fibers. Here, the identical fibers are considered only and the generated periodic RVE and mesh of composites reinforced by the spatially randomly distributed fibers with the aspect ratios of 15 and fiber volume fraction of 10% are shown in Fig. 2.

2.2. Governing equations of numerical homogenization

Consider a macrostructure with infinitesimal deformation that is heterogeneous at the micro-scale. When an RVE is analyzed for homogenization, what is essentially done is that the original heterogeneous material M in this RVE is replaced by a homogeneous effective material M*. Boundary conditions and the presence of body forces may strongly affect the stress and strain fields in the RVEs. The effective constitutive equations, however, are formulations that are intrinsic to materials, and should not depend on such external effects. Therefore, the boundary value problem for testing an RVE is to determine $\boldsymbol{u}(\boldsymbol{x})$ in the following case,

$$div(\boldsymbol{\sigma}^*) = 0 \tag{1}$$

so that subject to the appropriate boundary conditions on ∂V.

For the linear elasticity of composites, the macroscopic stress $\boldsymbol{\sigma}^*$ is a function of the macroscopic strain $\boldsymbol{\varepsilon}^*$ only, i.e. $\boldsymbol{\sigma}^* = \boldsymbol{\sigma}^*(\boldsymbol{\varepsilon}^*)$ without any inelastic variables and it is path-independent due to the elasticity of the constituents. Therefore, one immediately concludes that the macroscopic constitutive formulation is also purely elastic and takes the form of,

$$\boldsymbol{\sigma}^* = \boldsymbol{C}^*\boldsymbol{\varepsilon}^* \quad \text{and} \quad \boldsymbol{\varepsilon}^* = \boldsymbol{S}^*\boldsymbol{\sigma}^* \tag{2}$$

where $\boldsymbol{C}^*$ is the effective stiffness tensor and $\boldsymbol{S}^* = (\boldsymbol{C}^*)^{-1}$ is the effective compliance tensor. $\boldsymbol{\sigma}^*$ and $\boldsymbol{\varepsilon}^*$ are the effective stress and strain tensors, respectively, which are extracted using the volume averaging over an RVE and are recalled here:

$$\boldsymbol{\sigma}^* = \langle\boldsymbol{\sigma}\rangle = \frac{1}{V}\int_V \boldsymbol{\sigma} dV \quad and \quad \boldsymbol{\varepsilon}^* = \langle\boldsymbol{\varepsilon}\rangle = \frac{1}{V}\int_V \boldsymbol{\varepsilon} dV \tag{3}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain tensor in the RVEs, respectively, and calculated using the constitutive equations of the mechanical properties by applying the appropriate BCs. V is the volume of the RVE.