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Stress and deformation of multiple winding angle hybrid filament-wound thick cylinder under axial loading and internal and external pressure

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ABSTRACT

Multiple winding angle filament-wound (FW) structures may lead to more uniform strength for all filament layers, while there is no analysis method for deformation and stresses calculation of FW cylinder with variation wound angle. Deformation and stresses of a thick cylinder with multi-angle winding hybrid filament under axial loading and internal and external pressure were investigated. The deformation and stresses of each orthotropic unit of fiber layers, as well as longitudinal stress along the fiber direction, transverse stresses perpendicular to the fiber direction and shear stress in fiber layer were obtained analytically. The degenerated case of the model includes single winding angle FW cylinder, FW vessel with a liner, and sandwich pipe with one or multiple core layers. Numeric results of finite element models are present, which are very close to the theoretical results. The research shows that material utilization and working pressure can be increased by multi-angle FW procedure.

1. Introduction

High performance fiber wound pressure vessel is widely used in missile, rocket, satellite, deep diving and civilian container [1]. Winding filaments have both excellent mechanical performances and shortcomings. E-glass fiber has lower strength and elasticity, and high toughness; polymer has high strength and elasticity, and lower compressive strength; Carbon fiber has high tensile strength and elasticity, and lower fracture toughness. Hybrid filament wound (FW) structure can remedy the shortcomings and take the advantages. Thin walled FW shell was well studied [2,3], but 3-D analysis of hybrid FW thick cylinder under axial loading and internal and external pressure has seldom been investigated.

Modern computerized equipment allows for the production of multi-angle winding structures [4]. Using internal pressure and axial force, experiments under biaxial tensile stress ratios were carried out to investigate the performance of multi-angle FW structures [5]. Multi-angle wound structures exhibited better performance in resisting damage and greater advantages over pure angle-ply lay-ups [6]. Many experimental failure analyses [7–11] have been conducted for pipes with different winding angles, and an optimum winding angle of 55° has been noted for thin pipes subjected to internal pressure or biaxial loads with a hoop-to-axial stress ratio of 2:1. Researches show that stacking sequence has great influence for pressure capability [12,13], where stacking sequence can only be computed by 3-D analysis [14].

Higher pressure capability leads to thicker wall thickness (such as gun canal, deep diving container etc.), 3-D anisotropic constitutive relation and thick-walled shell theory should be taken into account in these analyses [15]. Base on 3-D anisotropic elasticity, Xia studied the stress and deformation of multi-layered FW composite pipes under internal pressure [16]. Takayanagi investigated the results of analytical and experimental studies on the rigidity and strength of FW fiber-reinforced composite pipes under internal pressure. The stiffness, deformation and strength of FW pipes wound at 30°, 45°, 55° and 70° were investigated [17]. FW fiber-reinforced pipes under internal pressure are usually subject to complicated loads involving biaxial or triaxial stresses [18–19]. The experimental results showed that the deformation and failure mechanisms of FW pipes are strongly dependent on winding angles [20]. Rosenow predicted the stress and strain response of pipes with winding angles varying from 15° to 85° by the classical laminated plate theory, and compared his predictions with experimental results [21].

Methods of analyzing fiber-dominated damage modes in composites have been well established for prediction of structural failures. However, composite pipes and vessels may also be subject to

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weepage failures caused by matrix damage [22]. The stress and strain fields in thick walled composite pipes with metal liner were obtained from an approximate closed form analytical solution by variational principles [23]. The fiber winding angle difference through thickness was calculated by a commercial FEA code, ABAQUS [24]. Bakaiyan analyzed multi-layered FW composite pipes under combined internal pressure and thermomechanical loading with thermal variations [25]. Ding established mixed state equation for axisymmetric problem of thick laminated closed cantilever cylindrical shells through weak formulation of equilibrium equations [26]. Huang analyzed laminated circular cylinders of materials with the most general form of cylindrical anisotropy under axially symmetric deformation [27].

Here, using 3-D orthotropic elasticity and axisymmetric thick-walled cylinder theory, deformation and stresses of a thick cylinder using multi-angle winding with different kind of filament with any number of layers under internal, external pressure and axial force is investigated analytically. To be convenient for strength assessment, longitudinal stress along fiber direction, transverse stresses perpendicular to the fiber direction and shear stress is present. As a degenerated case for any layer, isotropic constitutive relation is considered in the model for isotropic material or isotropic material in r–θ plane, which makes the model capable of simulating sandwich pipe and filament vessel with a liner. The relations between the distributions of stresses through thickness and winding angle under internal pressure are discussed, including composite vessel of uniform winding angle with a liner, a sandwich pipe with one core layer, and a multi-angle filament winding vessel respectively. Numeric results of FE models with the same parameters are given to validate the theory and the formulae in the paper.

2. Model and stresses in fiber layer

Consider a thick walled cylinder with an inner radius of r_0, and outer radius after alternate-ply FW of r_n. Each alternate-ply layer can be regarded as an orthotropic layer. Winding filaments are divided into n orthotropic layers with outer radius of r_i, ($i = 1, 2, \ldots, n$), with different kinds of filament and winding angles in each layer.

The cylinder is subjected to uniform internal pressure q_a and external pressure q_b. There may be axial force T_z along z axis. Only displacement in radial r direction and axial z direction occur (Fig. 1).

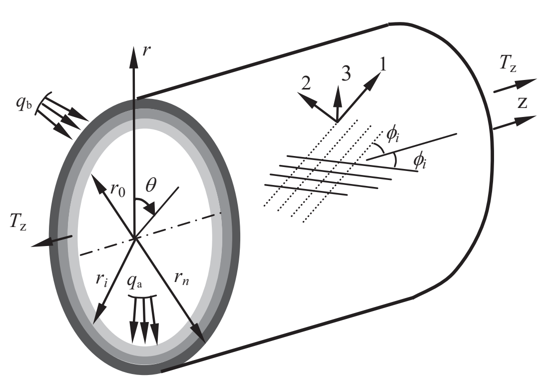

Fig. 1. Axisymmetric hybrid filament wound cylinder.

2.1. Basic definitions

Let $u_r(r)$, $u_\theta(r)$ and $u_z(r)$ denote the radial, circumferential and axial displacement, respectively. For layer (i), displacements and strains are defined as

$$\begin{aligned} &u_r = u_r^{(i)}(r), \quad u_\theta^{(i)}(r) = 0, \\ &u_z = u_z^{(i)}(z), \quad \varepsilon_r^{(i)} = \frac{du_r^{(i)}}{dr}, \quad (i = 1, 2, \ldots, n) \\ &\varepsilon_\theta^{(i)} = \frac{u_r^{(i)}(r)}{r}, \varepsilon_z^{(i)} = \frac{du_z^{(i)}}{dz} = \varepsilon_0 \end{aligned} \tag{1}$$

The equilibrium equation for each layer is

$$\frac{d\sigma_r^{(i)}}{dr} + \frac{\sigma_r^{(i)} - \sigma_\theta^{(i)}}{r} = 0, \quad (i = 1, 2, \ldots, n) \tag{2}$$

Superscript "(i)" is taken over 1, 2, …, n, corresponding to orthotropic layers outward.

2.2. On-axis stress–strain relation

The ply-oriented constitutive relationship can be expressed in matrix form as

$$\{\sigma\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = [C]\{\varepsilon\} \tag{3}$$

In which

$$[C] = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_2 & -\nu_{13}/E_3 & 0 & 0 & 0 \\ -\nu_{21}/E_1 & 1/E_2 & -\nu_{23}/E_3 & 0 & 0 & 0 \\ -\nu_{31}/E_1 & -\nu_{32}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix}^{-1} \tag{4}$$

For unidirectional orientation fiber composites, the fiber distributions are very similar in the 2 and 3 directions. Therefore, assuming transverse isotropy, and based on equivalent properties in the 2–3 plane for unidirectional material. We have

$$\begin{aligned} &E_2 = E_3, \quad G_{31} = G_{12}, \quad \nu_{12} = \nu_{13}, \\ &\nu_{21} = \nu_{31}, G_{23} = E_2/2/(1 + \nu_{23}) \end{aligned} \tag{5}$$

Then, the stiffness matrix Eq. (4) can be simplified as

$$[C]_{3\times3} = \frac{1}{1 - \frac{2\nu_{21}^2}{1-\nu_{23}}\frac{E_2}{E_1}} \begin{bmatrix} E_1 & \frac{\nu_{21}E_2}{1-\nu_{23}} & \frac{\nu_{21}E_2}{1-\nu_{23}} \\ & \frac{1-\nu_{21}^2E_2/E_1}{1-\nu_{23}^2}E_2 & \frac{\nu_{23}+\nu_{21}^2E_2/E_1}{1-\nu_{23}^2}E_2 \\ sym. & & \frac{1-\nu_{21}^2E_2/E_1}{1-\nu_{23}^2}E_2 \end{bmatrix} \tag{6}$$

$$C_{44} = \frac{E_2}{2(1 + \nu_{23})}, \quad C_{55} = C_{66} = G_{12}$$

2.3. Off-axis stress–strain relation

The strain energy can be calculated with on-axis stiffness constants or off-axis stiffness constants as

$$U = \frac{1}{2}\{\sigma\}^T\{\varepsilon\} = \frac{1}{2}\{\varepsilon\}^T[C]\{\varepsilon\} = \frac{1}{2}\{\bar{\varepsilon}\}^T[\overline{C}]\{\bar{\varepsilon}\} \tag{7}$$

In which $\{\boldsymbol{\varepsilon}\}$ and $\{\boldsymbol{\varepsilon}^-\}$, $\{\boldsymbol{C}\}$ and $\{\overline{C}\}$ are strain vector and stiffness matrix under on-axis coordinate system and off-axis coordinate system, respectively.