Composite Structures 125 (2015) 1–12

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Estimation of random field material properties for chopped fibre composites and application to vibration modelling

A.T. Fabro [a,*], N.S. Ferguson [a], J.M. Gan [b], B.R. Mace [c], S. Bickerton [c], M. Battley [b]

[a] ISVR, University of Southampton, Southampton, UK
[b] CACM, University of Auckland, Auckland, New Zealand
[c] Department of Mechanical Engineering, University of Auckland, Auckland, New Zealand

ARTICLE INFO

Article history:
Available online 30 January 2015

Keywords:
Fibre reinforcement
Random fields
Karhunen–Loeve
Vibration analysis
Experimental validation

ABSTRACT

Typically there is variability in the material and geometrical properties of fibre-reinforced composites and this variability is often spatially correlated. Numerical models can predict the response of such panels, but the spatially correlated nature of the variability must be represented within the model. However, characterising spatially correlated variability is problematic. In this study, data is first generated by an automated optical process; light transmissibility measurements are taken of a dry chopped strand mat panel. The consequent image is post-processed to describe the fibre density as a random field using the Karhunen–Loeve decomposition. Previous measurements have shown a strong correlation between the density of the mat and the tensile modulus, so the information is then used to infer the statistics of the stiffness matrix in a Finite Element model. The panel is then cut into beams, from which mobility measurements provide an ensemble of measured mobility and natural frequencies. Subsequent realisations of the random field and corresponding Finite Element model are then used to predict the statistics of the vibration response of the beams and compare well with measured statistics. The method provides an automated approach to the characterisation of spatial variability and hence the prediction of the statistics vibration response.

1. Introduction

Manufacturing processes often result in variability of properties compared to the nominal designed product. As the requirements for optimum design increase, it might be important to improve the prediction capability by incorporating such features. It is usual that the mechanical properties of composite structures are modelled by analytical models taking into account the mean properties in the structure, although they can exhibit great spatial variability.

The characterisation of the spatial variability becomes even more relevant when dealing with fibre reinforced composite materials, for instance, for which different fibre arrangements can affect the mechanical properties [1] and high material correlation is expected from the variability in physical features such as the fibre volume fraction [2]. Moreover, the use of a purely deterministic approach to model and, consequently, design structures using these materials can limit their applicability and require higher safety factors [3]. Even though the inclusion of spatial variability and uncertainties in mechanical models has received significant attention, for instance [4–10], it is very difficult to quantify the spatial variation of material properties by procedures involving, for instance, manual measurement. A few experimental procedures for characterisation of the spatial variability in fibre reinforced composite materials have been recently proposed, based on the volume fraction or fibre distribution.

Baxter [11] presented a methodology for characterising the randomness in a micromechanical model from a moving-window technique using micrographs of fibre reinforced composites. Gangadhar and Zehn [10] reviewed some methodologies to numerically simulate a number of different kinds of microstructural patterns in composite materials and subsequently include them within Finite Element (FE) models.

Guilleminot et al. [12,13] proposed a theoretical framework and experimental identification based on a probabilistic model of the elasticity tensor random field and also a probabilistic model based on the volume fraction, using ultrasonic scanning measurements. Mehrez et al. [14,15] present a technique of stochastic identification from limited experimental data, based on mobility frequency response functions measured at different points of a number of cantilever beams of woven composite material. The technique

* Corresponding author. Current address: Department of Mechanical Engineering, University of Brasília, Brasília, DF, 70910-900, Brazil. Tel.: +55 61 3107 5682.
E-mail address: fabro@unb.br (A.T. Fabro).

http://dx.doi.org/10.1016/j.compstruct.2015.01.036
0263-8223/

accounts for both aleatory, i.e. the inherent uncertainty of the material proprieties from sample to sample, and epistemic uncertainties, related to lack of sufficient experimental data, using a Hermite Polynomial Chaos [5] expansion of the random variables on the identified Karhunen–Loeve (KL) series.

In this work, an alternative method is presented for characterising the variability and its subsequent effect on the natural frequencies and mobilities of beams made of glass-fibre reinforced epoxy composite material composed of chopped strand mat (CSM). The material is randomly distributed over the panel and the distribution of Young's modulus is assumed to be isotropic. The spatial distribution correlation structure is not known *a priori* and firstly a measurement procedure, as proposed in [16], is applied to identify it. This initial step is performed using light transmissibility measurements taken from a composite panel, from a digital image captured by a DSLR camera. An empirical relationship between the pixel values to the fibre density along with a rule of mixtures is then used to estimate the spatial variation in the Young's modulus of elasticity. This information is used to estimate, from a single sample, the mean value and the covariance function of the spatial distribution. Furthermore, it is assumed that the fibre density random field [17] has a Gaussian distribution and a KL expansion is used to represent it in terms of a reduced set of independent Gaussian random variables [5,18]. The physical panel is then cut into beams, from which mobility and measurements are possible, providing an ensemble of measured mobility and natural frequencies. Results are finally compared with FE model approximations and simulations based upon the random field description.

2. Light transmission images from chopped strand mat panel

An apparatus, as proposed in [16], comprising a light box and a digital SLR camera has been assembled to capture high resolution light transmission images of reinforcement layers, from which information is extracted. The light box is illuminated by three fluorescent tubes and uses a simple plastic diffuser screen to help disperse light evenly. A Canon EOS 400D digital SLR camera was mounted on an aluminium extrusion frame attached to the light box, ensuring minimum relative movement between the camera and the illuminating surface. Optimum camera settings such as aperture and exposure length varied between reinforcement types and were chosen to give the best lighting conditions. Image distortion corrections are taken for two primary forms caused by the geometry of the lens, namely Barrel distortion, and by uneven illumination. The former is taken into account by software correction and the latter by subtracting off an image of the background gradient such that the amount of light entering L_e is the amount of light blocked L_b at pixel size, given as an integer number from 0–255 (grey scale), plus the amount of light transmitted L_t, i.e.

$$L_e = L_b + L_t. \tag{1}$$

Fig. 1 shows the fibre reinforcement before the infusion process with the epoxy matrix for the single and double layer panels, i.e. manufactured with one and two layers of CSM, respectively. The double layer panel has a larger thickness than that with one layer. An image of the light transmission for random CSM can be related to the area density, AW, in g/m^2, through an empirically derived polynomial fit [19]

$$AW(L_b) = k_0 + k_1 L_b + k_2 L_b^2 + k_3 L_b^3 + k_4 L_b^4, \tag{2}$$

where the fitting coefficients are given in Table 1. This approach neglects the fibre variability through the thickness.

The images were produced from pictures taken of two 450×290 mm panels, with single and double layers of the composite material. They were downsized to 63×40 pixels resolution, such that each pixel corresponding approximately to an area of 7×7 mm. Selection of this window size was based on a criterion for which the average pixelated area density had to match the macro-scale area density, i.e. $AW(L_b) \approx AW(L_b)$, where $\langle \cdot \rangle$ stands for the average [16]. This window size may correspond to the representative volume element (RVE) [20] size for the random CSM material.

It is expected that the pixel resolution from the original image greatly affects the spatial variability of the area density. The pixel size averages the property within that area, applying a smoothing effect, i.e. the finer the pixel resolution, the bigger the variability the image is able to capture. Baxter and Graham [11] applied a moving window technique for characterising the volume fraction of a fibre reinforced composite material and showed the corresponding changes on the correlation structure and probability density function (PDF) for different window sizes. Accurate stress calculations, though, might require a much finer description to properly represent the material constitutive matrix. An overview of various models of probabilistic homogenisation can be found at [21].

Assuming a constant thickness manufacturing process, the volume fraction V_f can be calculated as

$$V_f(AW) = \frac{AW}{\rho_f t} \tag{3}$$

where ρ_f is the fibre density and t is the thickness. The random distribution of the chopped strand implies that there is no preferential direction of fibre arrangement. Therefore, it is reasonable to assume isotropy for the Young's modulus and it is possible to relate Eq. (3) to a local Young's modulus E by a simple rule of mixtures [19,22], i.e.

$$\begin{aligned} E(E_L, E_T) &= \frac{3}{8} E_L + \frac{5}{8} E_T \\ E_L(V_f) &= E_f V_f + E_m (1 - V_f) \\ E_T(V_f) &= \left(\frac{V_f}{E_f} + \frac{1 - V_f}{E_m} \right)^{-1} \end{aligned} \tag{4}$$

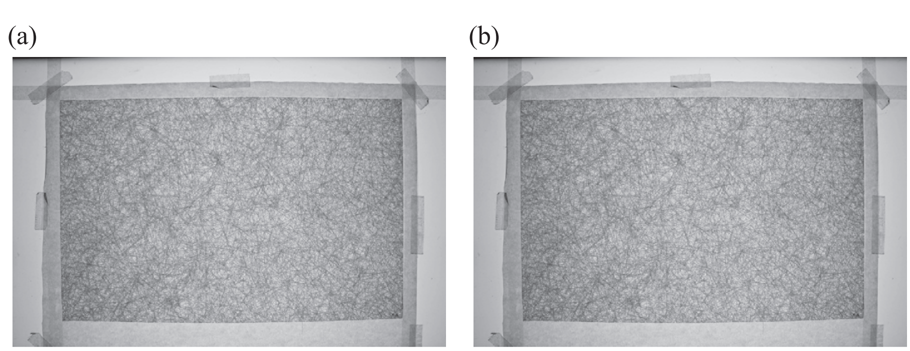

Fig. 1. Fibre reinforcement detail before the infusion process with the epoxy matrix for (a) the single layer panel and (b) the double layer panel.