Composite Structures 131 (2015) 927–938

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Modal density of sandwich panels based on an improved ordinary sandwich panel theory

Jingyong Han, Kaiping Yu *, Xiangyang Li, Rui Zhao

Department of Astronautic Science and Mechanics, Harbin Institute of Technology, No. 92 West Dazhi Street, Harbin 150001, China

ARTICLE INFO

Article history:
Available online 2 July 2015

Keywords:
Sandwich panels
Improved ordinary sandwich panel theory
Modal density
Boundary conditions

ABSTRACT

Modal density of sandwich panels with composite face sheets and orthotropic core is presented based on an improved ordinary sandwich panel theory. The governing equations are derived by applying the Hamilton's principle, where in-plane rigidity of the core is considered and the material and reference axes are not limited to be identical to each other. Modal density is obtained using wavenumber space integration, and the effects of boundary conditions on both modal density and mode counts are also studied. The accuracy of the proposed models is validated by comparing with the existing modal density expressions, piecewise shear deformation theory and finite element models. Parametric studies are performed in order to investigate the influence of the ply angle of face sheets and the core, in-plane rigidity of the core, transverse shear rigidity and boundary conditions on modal density. The proposed modal density formula has a wider application scope and will be beneficial to the predication of sound transmission and radiation of sandwich panels.

1. Introduction

Sandwich panels with composite face sheets have been increasingly applied in the structural design of aerospace, ships and other industries instead of metal structures in recent years due to their high stiffness-to-weight ratios, corrosion resistance, high damping and other features. Furthermore, the assessment of vibro-acoustic characters of sandwich panels must be made in most applications [1].

Statistical energy analysis (SEA) based on the power flow balance equation is the main theory in the field of structural vibro-acoustic characteristic analysis [2]. The predicted precision of the method mainly depends on the accuracy of statistical energy parameters, and one of the significant parameters is modal density which is defined as the average number of modes per unit frequency.

There have been some literatures about the analysis of modal density of sandwich panels. Clarkson and Ranky [3] derived a formulation for the modal density of honeycomb sandwich panels based on a simplified form of governing equations presented by Mead and Markus [4]. Moreover, they also obtained the modal density of honeycomb sandwich panels through a two-channel point mobility method experimentally. However, there was no mass correction of the impedance head in their experiment. Renji and Nair [5] presented a fourth-order governing equation of composite sandwich panels considering the transverse shear rigidity of the core based on the Mindlin theory and developed an expression of the modal density for sandwich panels with orthotropic laminate face sheets on the base of the governing equation derived previously. Soon afterwards, Renji [6] investigated the effects of both real and imaginary parts of driving point admittance in the measured mass correction. He also pointed that the imaginary part of the driving point admittance had a significant effect on the modal density of sandwich panels in the high frequency range, and that considering both real and imaginary parts of the measured admittance simultaneously could improve the accuracy of the measured modal density. However, the items including γ_{xz} and γ_{yz} were omitted without explanation in the process of derivation. Besides, the core was limited to be isotropic. It is important to emphasize that the governing equations of sandwich panels used by Clarkson [3] and Renji [5], which are used to derive the expression of modal density, are both fourth-order governing equations. Zhou and Crocker [7] developed an expression of the modal density of sandwich panels based on a sixth-order governing equation, in which the bending rigidity of face sheets was considered, and indicated that the bending rigidity of face sheets had a significant effect on the modal density of sandwich panels in the high frequency range. However, the expression of modal density was only suitable for sandwich panels with isotropic face sheets. Besides, Guillaumie [8] studied the vibroacoustic flexural properties of symmetric

---

* Corresponding author.
E-mail address: yukp@hit.edu.cn (K. Yu).

http://dx.doi.org/10.1016/j.compstruct.2015.06.039
0263-8223/© 2015 Elsevier Ltd. All rights reserved.

honeycomb sandwich with isotropic and laminate faces in terms of wavenumber modulus, modal density and mechanical impedance. However, the modal density formulas derived by Clarkson [3] and Renji [5] are directly used for sandwich panels with isotropic and laminate faces respectively. Cherif et al. [9] have made some experimental investigation on the modal density for thin and thick sandwich panels. He suggested that the input mobility method at low frequency and the wavenumber method at mid and high frequency should be combined in order to obtain better results. Ghinet and Atalla [10] also studied the numerical computation of the modal density for sandwich composite panels using the proposed 6th and 12th order relation of dispersion. Chronopoulos et al. [11] calculated the modal density of two dimensional composite orthotropic structures through the dispersion characteristics, which was predicted in the framework of Wave Finite Element method. Using the modal expansion technique and equivalent properties of the panel and beams, Mejdi and Atalla [12,13] developed models to predict vibroacoustic behavior of stiffened metallic and composite panels which can be used to quick estimate the modal density of stiffened panels. Finnveden [14] evaluated the modal density of the beam with arbitrary cross-section based on the waveguide-FEM. Bachoo and Bridge [15] derived an analytical expression for the modal density of fiber reinforced composite beams coupled in bending and torsion, where the effects of boundary conditions were neglected in order to simply the expression of the modal density. Xie et al. [16] developed an approximate model for the modal density of extruded panels, and the proposed is verified by an FE model.

The research of the sandwich panels modeling theory used for calculation of modal density has received much interest in the literatures for quite a long time, such as the classical laminated theory (CLT), Mindlin theory [17] considering transverse shear effect, high order shear deformation theory (HOSDT) [18], and piecewise shear deformation theory (PSDT) [19]. Review of various theories on the free vibration analysis of multilayered laminated composite and sandwich plates was presented by Sayyad and Ghugal [20]. The ordinary sandwich panel theory (OSPT) [21], as a layered-wise model, has considerable accuracy in predicting the natural frequencies of sandwich panels compared with the CLT, Mindlin theory and HOSDT. In addition, it is more convenient to obtain the modal density of sandwich panels than PSDT. However, it is noteworthy that OSPT ignores the in-plane rigidity of the core and restricts the consistency of the material and reference axes.

Furthermore, the effects of boundary conditions on the calculation of the modal density shall not be neglected. Bogomolny and Hugues [22] presented expressions for the mode counts of rectangular plates under standard boundary conditions. Xie et al. [23] investigated the effects of boundary conditions on both mode counts and the modal density systematically using wavenumber space integration technique based on the phase-closure principle [24]. In their research, general expressions for mode counts and modal density of one-dimensional and two-dimensional systems were proposed considering the effects of boundary conditions. They also demonstrated that the modal density depended on boundary conditions, and the effects of boundary conditions on both mode counts and the modal density were significant in the low frequency range. Farshidianfar et al. [25] proposed a modified wavenumber space integration method for calculating mode counts and modal density of circular cylindrical shells by applying more precise equation and considering boundary condition effects. Unfortunately, research on the modal density calculation of sandwich panels has not been conducted as far as the effects of boundary conditions concerned.

Due to a couple of shortcomings of the existing achievements discussed above, the aim of this present paper is to propose an improved ordinary sandwich panel theory (IOSDT), and to derive the modal density of sandwich panels using wavenumber space integration with the consideration of boundary condition effects.

This article is organized as follows: In Section 2, the governing equations are developed by applying the Hamilton's principle based on the IOSPT with both the in-plane rigidity of the core and the case that material and reference axes are different taken into account. Various simplified models are derived by simplifying the original model. In Section 3, modal densities of sandwich panels are developed on the basis of various models established in Section 2 through employing wavenumber space integration. Additionally, the effects of boundary conditions on both the modal density and mode counts are investigated based on the original governing equations obtained via IOSPT. In Section 4, numerical simulations are performed on several sandwich panels. The effectiveness of the proposed models is validated using the existed formulations, the method of counting modes and FEM models. Parametric studies are presented to determine the effects of ply angle of face sheets and core, in-plane rigidity of the core, transverse shear rigidity and boundary conditions on the modal density of sandwich panels. Summary and conclusions are presented in the sequel.

2. Governing differential equation

2.1. Basic assumptions

The research object of this article is symmetric sandwich panels, which are generally constituted of two thin, stiff, strong face sheets and a thick lightweight core with low stiffness and strength compared to face sheets located between the skins. The mid-plane of the sandwich panel is located in the *xoy* plane, as depicted in Fig. 1. In this research, we present an improved ordinary sandwich panel theory with the consideration of the in-plane rigidity of the core, and also considering that the core and face sheets materials are both orthotropic but with different material and reference axes. The following assumptions are made:

(1) The sandwich panel is symmetric about the mid-plane of the core;
(2) Classical thin plate theory is used for the face sheets since the skins are thin compared with the core;
(3) The normal stress in the thickness direction can be neglected. Thus, only antisymmetric deformation is considered;
(4) Considering the transverse shear deformation of the core, the in-plane rigidity of the core cannot be neglected although the core is relatively soft;
(5) The core and face sheets are bounded perfectly, and the mass of the adhesive at the face/core interface can be ignored.

Considering a sandwich panel with dimension $a \times b$, the *x*-direction is set along the *a* side of the panel. The thickness of the face sheet and core are *d* and *c*, respectively. The material parameters of the face sheet include elasticity modulus E_{f11}, E_{f22},

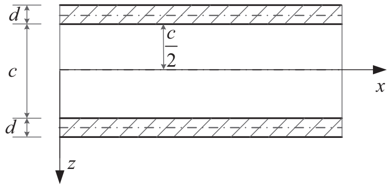

Fig. 1. A symmetric sandwich panel.