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Size-dependent effects on electromechanical response of multilayer piezoelectric nano-cylinder under electro-elastic waves

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ABSTRACT

Using the electro-elastic surface/interface model for nano-sized structures, an analytical model is proposed to investigate the dynamic electromechanical response of a multilayer piezoelectric nano-cylinder subjected to electro-elastic waves, and the interface energy effect on the stress and electric field is considered. To express the electro-elastic coupling field, the wave function expansion method is introduced. At the interface of each layer, the boundary conditions with consideration of interface energy are given. Some examples of a nano-cylinder with two or three layers are given to show the distribution of internal stress and electric field around the nano-cylinder under different surface/interface effects. It is found that softer surfaces/interfaces at the outermost layer are preferable to reduce the jump of dynamic stress. The effects of thickness and numbers of piezoelectric layers on the dynamic stress and electric field are also examined.

1. Introduction

Piezoelectric nano-structures (PNs) can harvest the ambient energy to produce electricity. Due to the outstanding nano-sized piezoelectric and semiconducting properties, they are finding lots of applications in powering nano devices and sensors in the field of medical science, defense technology, and environment/infrastructure monitoring. In recent years, numerous researchers have applied various methods to obtain the desired performance, and it is expected that this research will attract more and more attention in the following decades.

Compared with single-layer actuators, multilayer piezoelectric structures show good performance such as low power consumption, and are frequently used as active electronic components under severe environments. For example, switching from a single 0.5 or 1.0 μm layer of PZT to multiple 250-nm-thick layers would result in a 2/3 to 3/4 reduction in actuation voltage and an increase of 2–3 times in actuation force per area for a PZT switches and robotics [1]. In the past decade, lots of microelectromechanical systems (MEMs) made of multilayer piezoelectric composites have been investigated for numerous engineering applications [2–3]. Hauke et al. studied the internal mechanical stress and deflection of a multilayer cantilever actuator [4]. Yang and Xiang investigated the dynamic response of monomorph, bimorph, and multimorph actuators made of multilayer piezoelectric materials under a combined thermal-electro-mechanical load [5]. Dai et al. obtained the analytical solution of stresses in multilayer piezoelectric hollow structures [6]. Using a model of arbitrary multilayers on a substrate, Krantz and Gerken calculated the resonant bending-mode magnetoelectric (ME) coefficients of magnetostrictive–piezoelectric multilayer cantilevers, and it was found that ME coefficients strongly increase with cantilever thickness primarily due to increasing resonance frequencies [7]. An improved 1-D stress model for a single layer transformer in vacuum was used to model the multilayered transformer mounted on a printed circuit board, and a square pulse signal was amplified [8].

In self-powered nano-devices, a key component of energy harvesting is the one-dimensional piezoelectric nanostructures (PNs). Compared with two-dimensional PNs, one-dimensional PNs have three unique advantages: excellent piezoelectric effect, superior mechanical properties, and high sensitivity to small forces [9]. As a significant class of smart materials and structures, cylindrical piezoelectric structures have been frequently investigated. Based on one-dimensional nanofibrous materials for wearable electronics textiles applications, Gheibi et al. fabricated a one-step nano-generator, and the piezoelectric properties of fabricated composites were also evaluated on a self-made system as a function of frequency [10]. Recently, the finite element modeling and artificial neural network were presented to study the elastic buckling of smart lightweight column structures integrated with a pair of piezoelectric layers [11].

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For PNs, the size-dependent behavior is significantly related with the large surface/interface ratio to the volume. The presence of surface/interface can result in the premature failure. Since piezoelectric nanocomposites made of ceramics are very popular in practical engineering, they are susceptible to a brittle fracture that can lead to a catastrophic failure. To control the local elastic fields and minimize the stress concentration caused by the surfaces/interfaces in PNs, the distribution of stress concentration and electric field under different loadings should be addressed. To analyze the surface/interface effect, the electro-elastic surface/interface model was proposed [9] and widely used to study the response of PNs [12,13].

For multilayer piezoelectric transducers, the surface/interface effect becomes more evident due to the larger surface-to-volume ratio. It is important for us to understand the behavior resulting from surfaces/interfaces in piezoelectric nanostructures under the coupled electromechanical fields [14]. In electronic engineering, multilayer nano-devices are frequently used under severe environment conditions. The dynamic electro-elastic loading can simulate the severe conditions. To analyze the efficiency and durability of the newly developed multilayer piezoelectric nano-cylinder, the electromechanical response of this structure subjected to electro-elastic waves is studied in this paper. An electro-elastic surface/interface model is proposed to analyze the size-dependent effects on the distribution of stress and electric field inside the nano-cylinder. In numerical examples, the effects of surface/interface and the thickness of layers on the distribution of stress and electric field are analyzed in detail.

2. Problem description

A multilayer piezoelectric nano-cylinder is considered, as depicted in Fig. 1. The numbers of piezoelectric layers are denoted by $1,2,3,\ldots n$. The radii of piezoelectric layers are $a_1, a_2, \ldots a_n$. The radial thicknesses of each layer may be different, and are denoted by $h_1, h_2, \ldots h_n$. The elastic stiffness, piezoelectric constant, dielectric constant, and mass density of nano-cylinder are denoted by c_{44}^0, e_{15}^0, χ_{11}^0 and ρ^0. Those of ith piezoelectric layer are c_{44}^i, e_{15}^i, χ_{11}^i and ρ^i $(i = 1, 2, \ldots, n)$.

It is supposed that an anti-plane shear wave impinges on the multilayer nano-cylinder. Due to the nano-sized property, the surfaces/interfaces express significant effect on the strength and electric field inside the nano-cylinder. According to the electro-elastic surface/interface theory, the interface region has its own electromechanical properties, and can be regarded as a negligibly thin layer adhered to the adjacent solids. The surfaces/interfaces around the layers are denoted by Γ_i $(i = 1, 2, \ldots, n)$, as shown in Fig. 1. At the interfaces, the coupling of interfacial stress and electric displacement exists. The material properties of interfaces are different from the adjacent solids and denoted by c_{44}^{Si}, e_{15}^{Si}, χ_{11}^{Si} and ρ^{Si} $(i = 1, 2, \ldots, n)$.

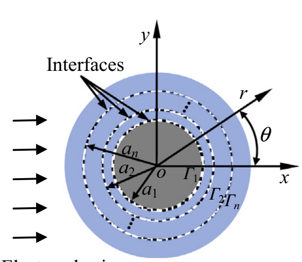

Fig. 1. A multilayer piezoelectric nano-cylinder subjected to electro-elastic waves.

3. Basic equations and solutions in different regions

3.1. Governing equations in the piezoelectric multilayer

When the anti-plane shear wave impinges on the multilayer nano-cylinder, the three-dimensional problem can be simplified, and only the anti-plane elastic displacement (u) and in-plane electric displacements (D_x, D_y) exist. The mechanically and electrically coupled constitutive equations can be written as

$$\sigma_{xz} = c_{44}\frac{\partial u}{\partial x} + e_{15}\frac{\partial \varphi}{\partial x}, \quad \sigma_{yz} = c_{44}\frac{\partial u}{\partial y} + e_{15}\frac{\partial \varphi}{\partial y}, \tag{1}$$

$$D_x = e_{15}\frac{\partial u}{\partial x} - \chi_{11}\frac{\partial \varphi}{\partial x}, \quad D_y = e_{15}\frac{\partial u}{\partial y} - \chi_{11}\frac{\partial \varphi}{\partial y}, \tag{2}$$

where σ_{jz} $(j = x, y)$ and φ are the shear stress and electric potential, respectively.

The governing equations in the multilayer nano-cylinder can be expressed as

$$c_{44}\nabla^2 u + e_{15}\nabla^2 \varphi = \rho \frac{\partial^2 u}{\partial t^2}, \tag{3}$$

$$e_{15}\nabla^2 u - \chi_{11}\nabla^2 \varphi = 0, \tag{4}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator.

By introducing another electro-elastic field $\psi = \varphi - \eta u$, Eqs. (3) and (4) can be decoupled as

$$\nabla^2 u = \frac{1}{c_{SH}^2}\frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$\nabla^2 \psi = 0, \tag{6}$$

where $\eta = e_{15}/\chi_{11}$, $c_{SH} = \sqrt{\mu^e/\rho}$ is the wave speed of electro-elastic waves in the piezoelectric media, and $\mu^e = c_{44} + \frac{e_{15}^2}{\chi_{11}}$.

3.2. Wave fields around the multilayer nano-cylinder

3.2.1. Electro-elastic coupling wave in the outermost layer

In the outermost layer (nth layer), two kinds of waves (incident waves and scattering waves from the next layer) exist. In the cylindrical coordinate system, the incident waves can be expanded as

$$u^{in} = u_0 e^{ik_n x} = u_0 \sum_{m=0}^{\infty} \varepsilon_m \mathrm{i}^m J_m(k_n r)\cos(m\theta), \tag{7}$$

where $\varepsilon_m = \begin{cases} 1, m = 0 \\ 2, m = 1, 2, 3, \ldots \end{cases}$, u_0 is the amplitude of the incident waves, $k_n = \omega/\sqrt{\mu_n/\rho_n}$ with $\mu_n = c_{44}^n + \frac{(e_{15}^n)^2}{\chi_{11}^n}$ being the electro-elastic wave number, and $J_m(\cdot)$ denotes the mth Bessel function of the first kind.

Similarly, the incident electric potential φ^{in} is expressed as

$$\varphi^{in} = \varphi_0 e^{\mathrm{i}(k_n x - \omega t)} = \varphi_0 \sum_{m=0}^{\infty} \varepsilon_m \mathrm{i}^m J_m(k_n r)\cos(m\theta), \tag{8}$$

where $\varphi_0 = \frac{e_{15}^n}{\chi_{11}^n} u_0$.

In the outermost (nth) layer, the general solutions of scattered field can be expressed as

$$u^{sc} = u_0 \sum_{m=0}^{\infty} a_m H_m(k_n r)\cos(m\theta), \tag{9}$$

$$\varphi^{sc} = \frac{u_0 e_{15}^n}{\chi_{11}^n} \sum_{m=0}^{\infty} a_m H_m(k_n r)\cos(m\theta) + \sum_{m=0}^{\infty} b_m r^{-m}\cos(m\theta), \tag{10}$$