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Free vibration of layered magneto-electro-elastic beams by SS-DSC approach

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ABSTRACT

Based on three-dimensional elasticity theory, semi-analytical solutions for free vibration of arbitrary layered magneto-electro-elastic beams are derived applying the state space approach (SSA) and discrete singular convolution (DSC) algorithm. The thickness direction of beams is chosen as the transfer direction in SSA, and the DSC is employed to discretize the length direction. Hence, the original partial differential equations are transformed into a state equation consisting of first-order ordinary differential equations. The application of DSC can implement various boundary conditions, which cannot be solved in the conventional SSA. Numerical examples are presented to study the method in details.

1. Introduction

Due to increasing applications of piezoelectric and piezomagnetic materials in smart structures, the problems associated with magneto-electro-elastic materials have attracted considerable attention recently [1–5]. There exhibit a coupling effect between electric and magnetic fields in these materials, which can be applied in smart structures. Applying these materials energy can be converted from one form to another, for example magnetic, electric and mechanical energy. Much research has been done on mechanical behavior of magneto-electro-elastic materials.

Pan [6] derived an exact three-dimensional solution of simply supported multilayered magneto-electro-elastic plate. Wang [7] applied the state space formulations [8,9] to study the bending of multi-layered magneto-electro-elastic rectangular plates. Chen and Lee [10] presented novel state space formulations for the static problem of transversely isotropic thermo-magneto-electro-elastic materials using a separation technique. Pan and Heyliger [11] found that some natural frequencies of a multi-field plate were identical to the ones of the corresponding elastic plate. Nonlinear free vibration of magneto-electro-elastic rectangular plates is studied by Razavi [12]. Wang [13] presented state vector approach of free-vibration analysis of magneto-electric-elastic hybrid laminated plates.

The state space approach was introduced into elasticity by Bahar [14] and showed high accuracy for laminated structures. Chen [15] applied the state-space-based differential quadrature (SS-DQM) to analyze angel-ply laminated beams. The SS-DQM was also used to study vibration of laminated plates [16] and cylindrical panels [17]. Annigeri [18] presented free vibration behavior of multiphase and layered magneto-electro-elastic beam. Li [19] studied free vibration of a functionally graded piezoelectric beam by state-space based differential quadrature.

Discrete singular convolution (DSC) algorithm was introduced by Wei [20] in 1999. As Wei's statement, singular convolutions are a special class of mathematical transformations which appear in many science and engineering problems, such as Hilbert, Abel and Radon transforms. It is the most convenient way to discuss the singular convolution in the context of the theory of distributions. It not only provides a rigorous justification for a number of informal manipulations in physical science and engineering, but also opens a new area of mathematics [21]. The theory of wavelets and frames can also be explained in the theory of distributions [22].

Wang [23] studied simply supported anisotropic rectangular plate by DSC method. Nonlinear static response of laminated composite plates is analyzed by using DSC method [24]. Ö Civalek [25] studied vibration of isotropic conical shells applying DSC. Vibration analysis of conical panels was presented by Ö Civalek [26] using DSC method. A four-node DSC for geometric transformation was applied to investigate vibration of arbitrary straight-sided quadrilateral plates [27].

Zhao et al. [28,29] analyzed the high frequency vibrations of plates and plate vibration under irregular internal support using DSC algorithm. Wan et al. [30] studied the unsteady incompressible flows using DSC. Ng et al. [31] presented a comparative accuracy of DCS and generalized differential quadrature methods

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for vibration analysis of rectangular plates. Yunshan [32] applied the DSC-Ritz method to study the free vibration of Mindlin plates. Lim et al. [33] proposed the DSC-Ritz method for high-mode frequency analysis of thick shallow shells. It is showed that the DSC algorithm works very well for the vibration analysis of plates, especially for high-frequency analysis. It is also concluded that the DSC algorithm had global methods' accuracy and local methods' flexibility for solving differential equations.

Giunta et al. [34] analyzed mechanical behavior of beams subjected to thermal loads via collocation with radical basis functions. The advantage is that it is easy to implement and provides higher order smoothness of solution, but the choice of RBFs' shape parameter affects accuracy greatly. Civalek [35] applied differential quadrature and harmonic differential quadrature to analyze buckling of thin isotropic plates and elastic columns. The various boundary conditions can be easily incorporated in differential quadrature and harmonic differential quadrature. Results obtained with HDQ method are more accurate than finite elements and finite differences and a coarser grid is used. Zhao et al. [36] presented DSC analysis of free-edged beams by iteratively matched boundary method. DSC algorithm had global methods' accuracy and presented good performance for high frequency analysis. However, it is difficult to incorporate some boundary conditions in DSC approach. The present method is not suitable for non-rectangular plates through geometric transformation because boundary conditions can not be easily incorporated in SS-DSC for non-rectangular plates.

In this paper, the hybrid semi-analytical elasticity method (SS-DSC) is introduced to the analysis of layered magneto-electro-elastic beams. The method can be applied to analyze arbitrary layered magneto-electro-elastic beams. The thickness direction of the laminates is treated as the transfer domain in the SSA, while the length direction is discretized by the DSC algorithm. Vibrations with various boundary condition can be studied by DSC algorithm. By applying the conventional transfer matrix method, a transfer relation between the state vectors on the lateral surfaces are established. Numerical examples are performed to validate the present method, and the accuracy and efficiency of this method is proved.

2. Basic equations

For a transversely isotropic magneto-electro-elastic medium in Cartesian coordinate system, the coupled constitutive equations can be written as [6]

$$\begin{aligned}\sigma_j &= C_{jk}S_k - e_{kj}E_k - q_{kj}H_k,\\ D_j &= e_{jk}S_k + \varepsilon_{jk}E_k + m_{jk}H_k,\\ B_j &= q_{jk}S_k + m_{jk}E_k + \mu_{jk}H_k,\end{aligned} \tag{1}$$

where σ_j denotes stress, D_j is electric displacement and B_j is magnetic induction. C_{jk}, ε_{jk} and μ_{jk} are the elastic, dielectric and magnetic permeability coefficient. e_{jk}, q_{jk} and m_{jk} are piezoelectric, piezomagnetic and magnetoelectric material coefficients.

In case of plane-stress state for a beam, the constitutive equations can be approximated to two-dimensional form as

$$\begin{aligned}\sigma_x &= \bar{C}_{11}S_{xx} + \bar{C}_{13}S_{zz} - \bar{e}_{31}E_z - \bar{q}_{31}H_z,\\ \sigma_z &= \bar{C}_{13}S_{xx} + \bar{C}_{33}S_{zz} - \bar{e}_{33}E_z - \bar{q}_{33}H_z,\\ \sigma_{zx} &= \bar{C}_{55}S_{xz} - \bar{e}_{15}E_x - \bar{q}_{15}H_x,\\ D_x &= \bar{e}_{15}S_{xz} + \bar{\varepsilon}_{11}E_x + \bar{m}_{11}H_x,\\ D_z &= \bar{e}_{31}S_{xx} + \bar{e}_{33}S_{zz} + \bar{\varepsilon}_{33}E_z + \bar{m}_{33}H_z,\\ B_x &= \bar{q}_{15}S_{xz} + \bar{m}_{11}E_x + \bar{\mu}_{11}H_x,\\ B_z &= \bar{q}_{31}S_{xx} + \bar{q}_{33}S_{zz} + \bar{m}_{33}E_z + \bar{\mu}_{33}H_z,\end{aligned} \tag{2}$$

in which

$$\begin{aligned}&\bar{C}_{11} = C_{11} - \frac{C_{12}^2}{C_{22}},\ \ \bar{C}_{13} = C_{13} - \frac{C_{12}C_{23}}{C_{22}},\ \ \bar{C}_{33} = C_{33} - \frac{C_{23}^2}{C_{22}},\ \ \bar{C}_{55} = C_{55},\\ &\bar{\mu}_{11} = \mu_{11},\ \ \bar{\mu}_{33} = \mu_{33} + \frac{q_{32}^2}{C_{22}},\ \ \bar{\varepsilon}_{11} = \varepsilon_{11},\ \ \bar{\varepsilon}_{33} = \varepsilon_{33} + \frac{e_{32}^2}{C_{22}},\\ &\bar{q}_{31} = q_{31} - \frac{C_{12}q_{32}}{C_{22}},\ \ \bar{q}_{33} = q_{33} - \frac{C_{23}q_{32}}{C_{22}},\ \ \bar{q}_{15} = q_{15},\\ &\bar{e}_{31} = e_{31} - \frac{C_{12}e_{32}}{C_{22}},\ \ \bar{e}_{33} = e_{33} - \frac{C_{23}e_{32}}{C_{22}},\ \ \bar{e}_{15} = e_{15},\\ &\bar{m}_{11} = m_{11},\ \ \bar{m}_{33} = m_{33} + \frac{e_{32}q_{32}}{C_{22}},\end{aligned} \tag{3}$$

The strain displacement relations are

$$S_{xx} = \frac{\partial u}{\partial x},\quad S_{zz} = \frac{\partial w}{\partial z},\quad S_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \tag{4}$$

where u and w are mechanical displacement in coordinate directions x and z.

The electric field vector can be expressed by the electric potential as follows:

$$E_x = -\frac{\partial \varphi}{\partial x},\quad E_z = -\frac{\partial \varphi}{\partial z}. \tag{5}$$

The magnetic field vector can be expressed by the magnetic potential as follows:

$$H_x = -\frac{\partial \psi}{\partial x},\quad H_z = -\frac{\partial \psi}{\partial z}. \tag{6}$$

3. State vector formulation

The state vector approach is based on the mixed formulation of solid mechanics in which $u, w, \sigma_z, D_z, B_z, \tau_{zx}, \phi$ and ψ are taken as basic unknowns. Following the process of state vector approach in elasticity and eliminating from the governing Eqs. (1)–(6), the field equations can be written in the following matrix form:

$$\frac{\partial \eta_1}{\partial z} = \mathbf{A}\eta_1, \eta_2 = \mathbf{B}\eta_1, \tag{7}$$

where η_1 is the basic unknown vector, which is called the state vector. η_2 is related to η_1 by Eq. (7).

$$\eta_1 = [u\ D_z\ B_z\ \sigma_z\ \tau_{zx}\ \phi\ \psi\ w]^T \eta_2 = [\sigma_x\ D_x\ B_x]^T, \tag{8}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & \rho\frac{\partial^2}{\partial t^2} - \alpha_1\frac{\partial^2}{\partial x^2} & -\alpha_4\frac{\partial}{\partial x} & -\alpha_5\frac{\partial}{\partial x} & -\alpha_3\frac{\partial}{\partial x}\\ 0 & 0 & 0 & 0 & -b_{21}\frac{\partial}{\partial x} & a_4 & a_5 & a_2\\ 0 & 0 & 0 & 0 & -b_{31}\frac{\partial}{\partial x} & a_5 & a_6 & a_3\\ 0 & 0 & 0 & 0 & -b_{11}\frac{\partial}{\partial x} & a_2 & a_3 & a_1\\ \frac{1}{C_{55}}, & -\beta_1\frac{\partial}{\partial x} & -\gamma_1\frac{\partial}{\partial x} & -\frac{\partial}{\partial x} & 0 & 0 & 0 & 0\\ -\beta_1\frac{\partial}{\partial x} & \beta_2\frac{\partial^2}{\partial x^2} & \beta_3\frac{\partial^2}{\partial x^2} & 0 & 0 & 0 & 0 & 0\\ -\gamma_1\frac{\partial}{\partial x} & \beta_3\frac{\partial^2}{\partial x^2} & \gamma_2\frac{\partial^2}{\partial x^2} & 0 & 0 & 0 & 0 & 0\\ -\frac{\partial}{\partial x} & 0 & 0 & \rho\frac{\partial^2}{\partial t^2} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{9}$$

$$\mathbf{B} = \begin{bmatrix} \alpha_1\frac{\partial}{\partial x} & \alpha_4 & \alpha_5 & \alpha_3 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \beta_1 & -\beta_2\frac{\partial}{\partial x} & -\beta_3\frac{\partial}{\partial x} & 0\\ 0 & 0 & 0 & 0 & \gamma_1 & -\beta_3\frac{\partial}{\partial x} & -\gamma_2\frac{\partial}{\partial x} & 0 \end{bmatrix}, \tag{10}$$